**NAME**

**COT 4210 Fall 2018 Midterm 2 Name: _______________**

**Total Points Available 68 Your Raw Score _______ Grade: M2: ____ Combined: _____**

*5* **1.** Write a Context Free Grammar for the language **L**, where
**L** $= \{ a^i b^j c^k \mid k = (i - j), \text{ if } i \geq j, \text{ else } k = 0 \}$. Hint: Splitting into two cases makes your job easier.

*3* **2.** Let **L1**, **L2** be Non-Regular CFLs; **R1**, **R2** be Regular; Answer is about **S** and there should be just one cell per row that has an **X**.

| ***Definition of S / Characterization of S*** | ***Always Regular*** | ***At worst CFL*** | ***Might not be CFL*** |
|---|---|---|---|
| ***S = L1 • L2*** | | | |
| ***S = R1 – L1*** | | | |
| ***S = R1 – R2*** | | | |
| ***S*** $\supseteq$ ***R1*** | | | |
| ***S*** $\subseteq$ ***R1*** | | | |
| ***S = L1*** $\cap$ ***R1*** | | | |

*2* **3.** Which of the following are correct definitions of an ambiguous grammar? In each case, **w** is a terminal string. Write **T**(true) or **F**(false) in the underlined area following each statement

There are two distinct parse trees for some string **w** derived by the grammar ____________

There are two distinct derivations of some string **w** derived by the grammar ____________

There are two distinct rightmost derivations of some string **w** derived by the grammar ____________

There are two distinct leftmost derivations of some string **w** derived by the grammar ____________

*4* **4**. Show that **Context-Free Languages** are closed under **Non-Empty-Left-Right Quotient with Regular Languages**. Non-Empty-Left-Right Quotient of a CFL **L** and a Regular Language **R**, both of which are over the alphabet **Σ**, is denoted **NELRQ(L, R)**), and defined as
**NELRQ(L, R)** $= \{ y \mid xyz \in L;\ x, z \in R;$ and $y \neq \lambda \}$.
That is, we select a non-empty substring **y** of **xyz** in **L**, provided **x** and **z** are both in the Regular Language **R**.
You may assume substitution **f(a) = {a, a'}**, and homomorphisms **g(a) = a'** and **h(a) = a, h(a') =** $\lambda$.
Here $a \in \Sigma$ and **a'** is a distinct new character associated with each $a \in \Sigma$. No justification is required.

**NELRQ(L, R)** = __________________________________________________

*12* **5.** Consider some language **L**. For each of (a) and (b), and for each of the three possible complexities of **L**, indicate whether this is possible (**Y** or **N**) and present evidence. Recall that
**max(A) = { w | w ∈ A and for no x≠λ does wx ∈ A }**
If you answer **Y**, you must provide an example language **A** and the resulting **L**. In the case of part (b) you must also present a homomorphism **σ**. If you answer **N**, state some known closure property that reflects a bound on the complexity of **L**. Note: I did the first of each of the three parts for you.

**a.)** **L = max(A)** where **A** is context-free, not regular.

Can **L** be Regular**?** Circle **Y** or **N.**

If yes, show **A** and argue **max(A)** is Regular; if no, why not?

**YES. Let A = { $a^i$ $b^j$ | i, j > 0 and j >i }**

**L = max(A) = ∅, a regular set, as every string in A can be extended with more b's.**

Can **L** be a non-regular CFL? Circle **Y** or **N.**

If yes, show **A** and argue **max(A)** is a CFL; if no, why not?

Can **L** be more complex that a CFL? Circle **Y** or **N.**

If yes, show **A** and argue **max(A)** is not a CFL; if no, why not?

**b.)** Let **σ be** a homomorphism from **Σ** into regular languages, such that, for each **a∈Σ**, **σ(a) = $w_a$**, for some string **$w_a$**. Let **A** be a context free, non-regular language and let **L = σ(A).**

Can **L** be Regular? Circle **Y** or **N**.

If yes, show **A** and **σ**, and argue **σ(A)** is Regular; if no, why not?

**YES. Let A = { $a^n$ $b^n$ | n > 0 }**

**Define σ(a) = λ and σ(b) = λ**

**L = σ (A) = { λ }, a regular set.**

Can **L** be a non-regular CFL? Circle **Y** or **N.**

If yes, show **A** and **σ**, and argue **σ(A)** is a CFL; if no, why not?

Can **L** be more complex that a CFL? Circle **Y** or **N.**

If yes, show **A** and **σ**, and argue **σ(A)** is not a CFL; if no, why not?

***10*** **6.** Use the Pumping Lemma for CFLs to show that the following language **L** is not Context Free.
**L = { $a^n$ $b^{sum(1..n)}$ | n>0 }**. Here **sum(1..n)** = $\sum_1^n i$.
Be explicit as to why each case you analyze fails to be an instance of **L** and, of course, make sure your cases cover all possible circumstances. I have done the first two steps for you.

***ME: Assume L is Context Free***
***PL: Provides a whole number N>0 that is the value associated with L based on the Pumping Lemma***
***ME:***

*12* **7.** Present the **CKY** recognition matrix for the string **aababb** assuming the Chomsky Normal Form grammar, **G = ( { S, T, U, V, W, A, B }, { a, b }, R, S )**, specified by the rules **R**:.

| | |
|---|---|
| **S →** | **A T \| B U** |
| **T →** | **b \| B S \| A V** |
| **U →** | **a \| A S \| B W** |
| **V →** | **T T** |
| **W →** | **U U** |
| **A →** | **a** |
| **B →** | **b** |

| | **a** | **a** | **b** | **a** | **b** | **b** |
|---|---|---|---|---|---|---|
| **1** | | | | | | |
| **2** | | | | | | |
| **3** | | | | | | |
| **4** | | | | | | |
| **5** | | | | | | |
| **6** | | | | | | |

**A little help**

| **Non-Terminal** | **First Symbol in Rules** | **Second Symbol in Rules** |
|---|---|---|
| **S** | **None** | **T → B S ; U → A S** |
| **T** | **V → T T** | **S → A T ; V → T T** |
| **U** | **W → U U** | **S → B U ; W → U U** |
| **V** | **None** | **T → A V** |
| **W** | **None** | **U → B W** |
| **A** | **S → A T ; T → A V ; U → A S** | **None** |
| **B** | **S → B U ; T → B S ; U → B W** | **None** |

Is **aababb** in **L(G)?** ____________

What is the order of execution of this approach to determine if some **w**, **|w| = N**, is in **L?** _________

What is the algorithmic strategy, e.g., greedy, divide and conquer, dynamic programming, backtracking or randomized, associated with this **CKY** algorithm called? ____________________________

**8.** Consider the CFG **G = ( { S, A, B } , { a , b } , R , S )** where **R** is:
**S → SAb | AbBa**
**A → aS | a**
**B → bS | b**
In the PDA you create below, you may (and are encouraged) to use a transition diagram where transitions have arcs with labels of form **a, α → β** where **a** ∈ **Σ** ∪ {**λ**}, **α, β** ∈ **Γ***. Note: This just means that you can use extended stack operations that push or pop arbitrary length strings.

*6* **a.)** Present a pushdown automaton that parses the language **L(G)** using **either** a top down or bottom up strategy.
**INITIAL CONTENTS OF STACK = ______**

*1* **b.)** Now, using the notation of **ID**s (Instantaneous Descriptions, **[q, x, z]**), describe how your PDA in (a) accepts strings generated by **G**.

**9.** Demonstrate the steps associated with a Reduced, then Chomsky Normal Form grammar.

*3* **a.)** Consider the context-free grammar **G1 = ( { S, A, B }, { 0, 1 }, R1, S )**, where **R1** is:
**S → AB**
**A → 0A0 | λ**
**B → 1B1 | λ**

Remove all **λ**-rules, except possibly for a start symbol, creating an equivalent grammar **G1'**. Show **all** rules.
***Nullable = {*** ***}***

***3*** **b.)** Consider the context-free grammar **G2 = ( { S, A, B }, { 0, 1 }, R2, S )**, where **R2** is

**S → AB | B**
**A → 1A0 | 10**
**B → A | AA**

Remove all **unit** rules, creating an equivalent grammar **G**2'. Show **all** rules.

***Unit(S) = { }; Unit(A) = { }; Unit(B) = { }***

***3*** **c.)** Consider the context-free grammar **G3 = ( { S, A, B }, { 0, 1 }, R3, S )**, where **R3** is

**S → AB | BB**
**A → 1A0**
**B → 0B1 | 01**

Remove all non-productive non-terminals, creating an equivalent grammar **G**3'. Show **all** rules.

***Productive = { }; Unproductive = { }***

***4*** **d.)** Consider the reduced context-free grammar **G4 = ( { S, A, B }, { 0, 1 }, R4, S )**, where **R4** is

**S → AABB**
**A → 1B0 | 10**
**B → 0A1 | 01**

Convert to an equivalent Chomsky Normal Form grammar **G**4'. Show **all** rules.