

# Assignment # 7.1 Sample

1. For each of the following languages, either provide a grammar to show it is a CFL or employ the Pumping Lemma to show it is not

a.)  $L = \{ a^i b^j \mid j > 2 \cdot i \}$

b.)  $L = \{ a^n b^{n!} \mid n > 0 \}$

# Assignment # 7.2 Sample

2. Consider the context-free grammar  $G = ( \{ S \}, \{ a, b \}, S, P )$ , where  $P$  is:

$$S \rightarrow S a S b S \mid S b S a S \mid S a S a S \mid a \mid \lambda$$

Provide the first part of the proof that

$$L(G) = L = \{ w \mid w \text{ has at least as many } a\text{'s as } b\text{'s} \}$$

That is, show that  $L(G) \subseteq L$

To attack this problem we can first introduce the notation that, for a syntactic form  $\alpha$ ,  $\alpha_a =$  the number of  $a$ 's in  $\alpha$ , and  $\alpha_b =$  the number of  $b$ 's in  $\alpha$ . Using this, we show that if  $S \Rightarrow^* \alpha$ , then  $\alpha_b \leq \alpha_a$  and hence that  $L(G) \subseteq L$ :

A straightforward approach is to show, inductively on the number of steps,  $i$ , in a derivation, that, if  $S \Rightarrow^i \alpha$ , then  $\alpha_b \leq \alpha_a$ .