Assignment # 9.1a Key

1. Use quantification of an algorithmic predicate to estimate the complexity (decidable, re, co-re, non-re) of each of the following, (a)-(d):

a)HasDip = $\{ f \mid$ for some x and y, $y > x$, $f(x)\downarrow$, $f(y)\downarrow$ and $f(y) < f(x) \}$

$\exists \langle x,y,t\rangle[STP(f,x,t)\ \&\ STP(f,y,t)\ \&\ (y>x)\ \&\ (VALUE(f,y,t) < (VALUE(f,x,t)\ )]$

RE

Assignment # 9.1b Key

b)HasMax = { f | for some x, where f(x)↓, f(y) < f(x), whenever f(y)↓ }

$\exists\langle x,t\rangle\ \forall\langle y,s\rangle[STP(f,x,t)\ \&\ (STP(f,y,s) \rightarrow (VALUE(f,y,s) < (VALUE(f,x,t))\ ]$

Non-RE, Non-Co-RE

Assignment # 9.1c Key

c) NotLarge = { f | if $x \in Range(f)$ then $x<100$ }

$\forall <y,t>$ [STP(f,y,t) → (VALUE(f,y,t)<100)]

Co-RE

Assignment # 9.1d Key

ZeroStart = { <f,x> | if $x<100$ and $f(x)\downarrow$ in fewer than 100 steps, then $f(x)=0$ }

$\forall x_{x<100}$ [STP(f,x,99)) $\rightarrow$ (VALUE(f,x,t)=0)]

Could also state as

(x<100 & STP(f,x,99)) $\rightarrow$ (VALUE(f,x,t)=0)

REC

Assignment # 9.2 Key

1. Let sets A be recursive (decidable) and B be re non-recursive (undecidable).
   Consider C = { z | max(x,y), where x ∈ A and y ∈ B }. For (a)-(c), either show sets A and B with the specified property or demonstrate that this property cannot hold.

a) Can C be recursive?

YES. Consider A = ***N***. B = Halt. C = { x | x ≥ min(Halt) }

As Halt is non-empty, it has a min value, MIN, even if we don't know what that value is. No value in C can be less than MIN and all values ≥ MIN are in C as A contains every possible value in ***N.***

Assignment # 9.2b Key

b) Can C be re non-recursive?

YES. Consider A = {0}. B = Halt. C = Halt as every value, x, is max(x,0).

Thus, C is clearly re non-recursive.

Assignment # 9.2c Key

c) Can C be non-re?

No. Can enumerate C as follows.

First if A is empty then C is empty and so RE by definition.

If A is non-empty then A is enumerated by some algorithm f_A as recursive sets are RE.

As B is non-recursive RE, then it is non-empty and enumerated by some algorithm f_B.

Define f_C by $f_C(<x,y>) = \max(f_A(x),f_B(y))$. f_C is clearly an algorithm as it is the composition of algorithms. The range of f_C is then $\{ z \mid \max(x,y)$, where $x \in A$ and $y \in B \}$ = C and so C must be RE.