Assignment # 5.1

1. For each of the following, prove it is not regular by using the Pumping Lemma or Myhill-Nerode. You must do at least one of these using the Pumping Lemma and at least one using Myhill-Nerode.

a. $L = \{ a^i b^j \mid i > 2*j \}$

b. $L = \{ a^{f(n)} \mid f(0) = 2; f(i+1) = f(i)^2 \}$

c. $L = \{ a^{g(i)} \mid g(i) = i^3 \}$

Assign # 5.1a,b Answer

1a. $\mathbf{L = \{ a^i b^j \mid i > 2*j \}}$ using P.L.

1. Assume that **L** is regular
2. Let **N** be the positive integer given by the Pumping Lemma
3. Let **s** be a string $\mathbf{s = a^{2N+1}b^N} \in \mathbf{L}$
4. Since $\mathbf{s} \in \mathbf{L}$ and $\mathbf{|s| \geq N}$, **s** is split by PL into **xyz**, where $\mathbf{|xy| \leq N}$ and $\mathbf{|y| > 0}$ and for all $\mathbf{i \geq 0}$, $\mathbf{xy^iz} \in \mathbf{L}$
5. We choose $\mathbf{i = 0}$; by PL: $\mathbf{xy^0z = xz} \in \mathbf{L}$
6. Thus, $\mathbf{a^{2^\wedge N+1-y|}\ b^N}$ would be in **L**. But then, there would be equal or fewer than twice as many **a**'s as **b**'s, meaning $\mathbf{a^{2^\wedge N+1-|y|}\ b^N}$ cannot be in **L.**
   This is a contradiction; therefore L is not regular ■

1b. $\mathbf{L = \{ a^{f(n)} \mid\ f(0) = 2;\ f(i+1) = f(i)^2 \}}$ using P.L.

1. Assume that **L** is regular
2. Let **N** be the positive integer given by the Pumping Lemma
3. Let **s** be the string $\mathbf{s = a^{f(N)}} \in \mathbf{L}$
4. Since s $\in$ L and $\mathbf{|s| \geq N}$ (actually $\mathbf{> N}$), **s** is split by PL into **xyz**, where $|\mathbf{xy| \leq N}$ and $\mathbf{|y| > 0}$ and for all $\mathbf{i \geq 0}$, $\mathbf{xy^iz} \in \mathbf{L}$
5. We choose $\mathbf{i = 2}$; by PL: $\mathbf{xy^2z = xyyz} \in \mathbf{L}$
6. Thus, $\mathbf{a^{f(N)+|y|}}$ would be in L, but it's not since $\mathbf{f(N)+|y| \leq f(N)+N < f(N)+f(N) < f(N)^2}$ when $\mathbf{N>0}$. But the next value in the sequence is $\mathbf{f(N)^2}$, so $\mathbf{a^{f(N)+|y|} \notin L}$
7. This is a contradiction; therefore **L** is not regular ■

Sample Assign # 5.1c Answer

1c. $\mathbf{L = \{ a^{g(i)} \mid g(i) = i^3 \}}$ using P.L.

1. Assume that **L** is regular
2. Let **N** be the positive integer given by the Pumping Lemma
3. Let **s** be the string $\mathbf{s = a^{N\text{^}3}} \in \mathbf{L}$
4. Since **s** $\in$ **L** and $\mathbf{|s| \geq N}$, **s** is split by PL into **xyz**, where $\mathbf{|xy| \leq N}$ and $\mathbf{|y| > 0}$ and for all $\mathbf{i \geq 0}$, $\mathbf{xy^iz} \in \mathbf{L}$
5. We choose $\mathbf{i = 2}$; by PL: $\mathbf{xy^2z = xyyz} \in \mathbf{L}$
6. Thus, $\mathbf{a^{N\text{^}3+|y|}}$ would be in **L**.
   However, $\mathbf{N^3+|y| \leq N^3+N < N^3+N+1 < N^3 + 3N^2 + 3N+1 = (N+1)^3}$ **so** $\mathbf{a^{N\text{^}3+|y|} \notin L}$
7. This is a contradiction; therefore L is not regular ■

Assign # 5.1 Answer

1a. $\mathbf{L = \{ a^i b^j \mid i > 2*j \}}$ using M.N.
We consider the collection of right invariant equivalence classes $\mathbf{[a^i]}$, $\mathbf{i \geq 0}$.
It's clear that $\mathbf{a^i\ b^{2i+1}}$ is in the language, but $\mathbf{a^j\ b^{2i+1}}$ is not when $\mathbf{j>i}$ as $\mathbf{2i+1 < 2j}$.
This shows that there is a separate equivalence class $\mathbf{[a^i]}$ induced by $\mathbf{R_L}$, for each $\mathbf{i \geq 0}$.
Thus, the index of $\mathbf{R_L}$ is infinite and Myhill-Nerode states that $\mathbf{L}$ cannot be Regular. ■

1b. $\mathbf{L = \{ a^{f(n)} \mid f(0) = 2; f(i+1) = f(i)^2 \}}$ using M.N.
We consider the collection of right invariant equivalence classes $\mathbf{[a^{2\wedge(2\wedge i)}]}$, $\mathbf{i \geq 0}$.
It's clear that $\mathbf{a^{(2\wedge(2\wedge i))\wedge 2} = a^{2\wedge(2\wedge i)*2\wedge(2\wedge i)} = a^{2\wedge(2\wedge i\ +\ 2\wedge i)}} = \mathbf{a^{2\wedge(2*2\wedge i)}} = \mathbf{a^{2\wedge(2\wedge(i+1))}}$ is in $\mathbf{L}$.
$\mathbf{a^{2\wedge(2\wedge j)}\ a^{2\wedge(2\wedge i)} = a^{2\wedge(2\wedge j\ +\ 2\wedge i)} < a^{2\wedge(2\wedge j\ +\ 2\wedge j)}} = \mathbf{a^{2\wedge(2\wedge j)*2\wedge(2\wedge j)}} = \mathbf{a^{(2\wedge(2\wedge j))\wedge 2}}$ when $\mathbf{j>i}$.
But, $\mathbf{a^{(2\wedge(2\wedge j))\wedge 2}}$ is the shortest string longer than $\mathbf{a^{2\wedge(2\wedge j)}}$ in $\mathbf{L}$ and so $\mathbf{a^{2\wedge(2\wedge j)}\ a^{2\wedge(2\wedge i)}}$ is not in $\mathbf{L}$.
This shows that there is a separate equivalence class $[\mathbf{a^{2\wedge(2\wedge i)}}]$, induced by $\mathbf{R_L}$, for each $\mathbf{i \geq 0}$.
Thus, the index of $\mathbf{R_L}$ is infinite and Myhill-Nerode states that $\mathbf{L}$ cannot be Regular. ■

1c. $\mathbf{L = \{ a^{g(i)} \mid g(i) = i^3 \}}$ using M.N.
We consider the collection of right invariant equivalence classes $\mathbf{[a^{i\wedge 3}]}$, $\mathbf{i \geq 0}$.
It's clear that $\mathbf{a^{(i+1)\wedge 3} = a^{i\wedge 3}a^{3i\wedge 2+3i+1}}$ is in the language, but $\mathbf{a^{j\wedge 3}a^{3i\wedge 2+3i+1}} < \mathbf{a^{(j+1)\wedge 3}}$ when $\mathbf{j > I}$ and so is no in $\mathbf{L}$.
This shows that there is a separate equivalence class $\mathbf{[a^{i\wedge 3}]}$ induced by $\mathbf{R_L}$, for each $\mathbf{i \geq 0}$.
Thus, the index of $\mathbf{R_L}$ is infinite and Myhill-Nerode states that $\mathbf{L}$ cannot be Regular. ■

Assign # 5.2

2. Write a regular (right linear) grammar that generates the set of strings denoted by the regular expression (00 + 010 + 001)*.

G = ({S,T,U,V}, {0,1}, S, P)

P:

$$
\begin{aligned}
S &\rightarrow 0T \mid \lambda \\
T &\rightarrow 0S \mid 1U \mid 0V \\
U &\rightarrow 0S \\
V &\rightarrow 1S
\end{aligned}
$$

Assign # 5.3

Present a Mealy Model finite state machine that reads an input **x** ∈ **{0, 1}*** and produces the binary number that represents the result of adding binary **10101** to **x** (assumes all numbers are positive, including results). Note: The binary number is read from least to most significant bit.

Answer