COT 3100 10/31/24 10^{30} am

(1) Carefully read through all typed Counting Notes

(2) Last Counting Lec

(3) Posted HS → Due 4 days after Proj 1 → Counting, Probability

(4) Posted Grade Calculation - plug in own values formula, put tentative grade lines.

$S = \{1, 2, 3, \ldots, 30\}$

(a) How many subsets of S have 5 elements? $\binom{30}{5}$

(b) How many of the subsets in part (a) have a minimum element NOT equal to 5?

$\binom{30}{5} - [$ # subsets w/ min elem 5 $]$

5, —, —, —, —

must choose 4 more $\{6, 7, \ldots, 30\}$

25

$\binom{30}{5} - \binom{25}{4}$

(c) How many subsest of S w/size 5 have a minimume element < 5? $\{5, 6, 7, \ldots, 30\}$

$\binom{30}{5} - [$ subset of S of size 5 w/ min element 5 or more $]$

$\binom{30}{5} - \binom{26}{5}$

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$$

$2n$ items n RED n BLUE

How many ways to select n of these $2n$ items? $\longrightarrow \binom{2n}{n} = RHS$ ✓

Red	Blue	# ways
0	n	$\binom{n}{0}\binom{n}{n}$
1	$n-1$	$\binom{n}{1}\binom{n}{n-1}$
2	$n-2$	$\binom{n}{2}\binom{n}{n-2}$
3	$n-3$	$\binom{n}{3}\binom{n}{n-3}$
⋮		
i	$n-i$	$\binom{n}{i}\binom{n}{n-i}$
⋮		
$n-1$	1	$\binom{n}{n-1}\binom{n}{1}$
n	0	$\binom{n}{n}\binom{n}{0}$

ways to choose 0 red out of n → # ways to choose n blue out of n

$$\sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i}$$

$$= \sum_{i=0}^{n} \binom{n}{i}\binom{n}{i}$$

$$= \sum_{i=0}^{n} \binom{n}{i}^2 = LHS$$

COMPLICATED → count permutations

restriction is consonants when scanned left to right must be in alphabetical order.
Same has to be true about the vowels

C A C D E I L O M P T
— ↑ — — ↑ ↑ — ↑ — — — —

Idea: Select 4 slots out of 11 vowels

$\binom{11}{4}$

C A C D E I L O M P T
↑ ↑ ↑ ↑

3O, 4M, 5B

ways to place in row so that no 2 birch trees are next to each other

— — — — — — — — — — — — —

_ O B O B O _ M _ M B M B M B

8 gaps choose 5 of them to place Birch trees!

$\binom{8}{5}$ place birches

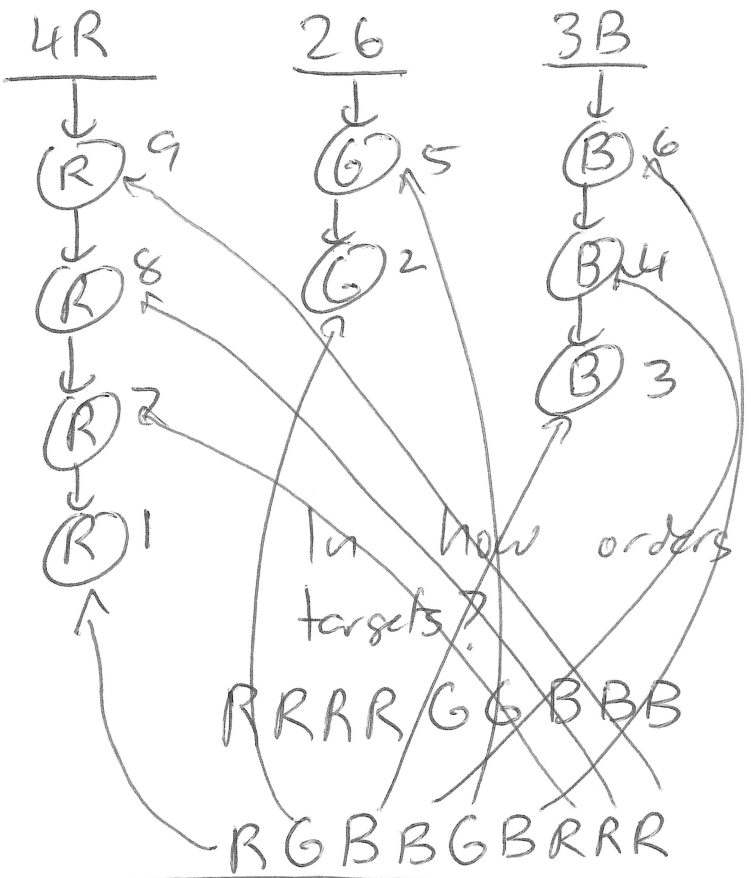
→ # orderings of Os, Ms = $\binom{7}{4}$, choose 4 slots out of 7 for the Maple trees

final ans = $\binom{8}{5} \times \binom{7}{4} = \binom{8}{3}\binom{7}{3} = \frac{8\times7\times6}{1\times2\times3} \times \frac{7\times6\times5}{1\times2\times3}$

$\binom{n}{k} = \binom{n}{n-k}$ $\binom{8}{3} = \frac{8!}{3!5!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{1\times2\times3\times5\times4\times3\times2\times1}$ $= 56 \times 35$

$= 1960$

Target Shooting



Shoot all 9 targets, but must always choose the bottom most target on a string

In how orders could I shoot the targets?

$$\frac{9!}{4!\,2!\,3!}$$

Spider

8 legs. 8 socks & shoes, Sock for each leg must go on before the shoe for that leg. But Shoe Leg_2 can be put on before Sock Leg_1.

A A B B C C D D E E F F G G H H

leg1 leg2 leg3 leg4

$$\frac{16!}{(2!)^8} = \frac{16!}{2^8}$$

B A H B C D F D C H A E F E G G

Sock 2, Sock 1, Sock 8, Shoe 2, Sock 3, Sock 4, Sock 6, Shoe 4 etc.

Bowling Tournament

A B

5th vs 4th → L → 5th B

↓ W C

A vs 3rd → L → 4th C

A ↓ W D

vs 2nd → L → 3rd A

D D ↓ W E

1st ← W vs 1st → L → 2nd E

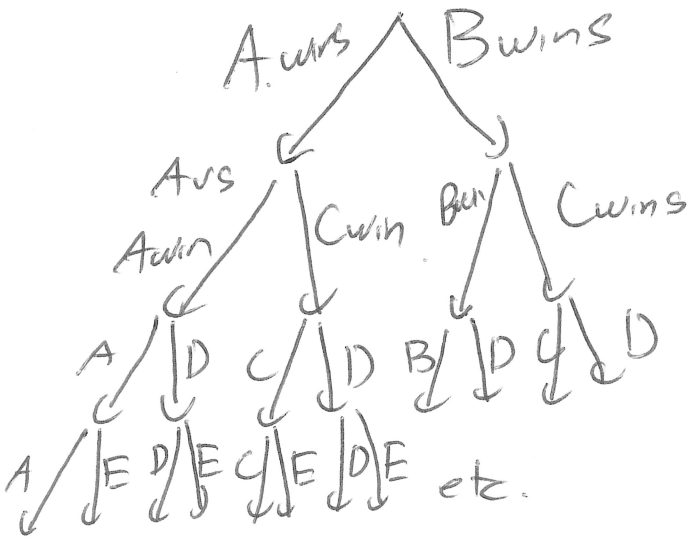
Order: D, E, A, C, B

2 choices 5th
2 choices 4th
2 choice 3rd
2 choice 2nd
x | choice 1st

$2^4 = 16$

4 matches, each match has 2 possible outcomes, any difference in outcome must lead to different finishing positions, so # finish orders must be 2^4, the # of possible outcomes of the games

LLWL



$A + B + C + D = 98$

A, B, C, D are positive ODD integers
How many solutions?

$A = 2a+1$, a non-neg int
$B = 2b+1$ b non-neg int
$C = 2c+1$ c " "
$D = 2d+1$ d " "

$(2a+1) + (2b+1) + (2c+1) + (2d+1) = 98$, good non neg ints

$2(a+b+c+d) = 94$

$a+b+c+d = 47$ } combos w/ repetion

$n = 47$
$r = 4$

$$\binom{47+4-1}{4-1} = \binom{50}{3}$$

How many permutations of 5 As, 5 Bs, 5 Cs are there such that there are no As in 1st 5 letters, no Bs in the second 5 letters, no Cs in last 5 letters?

$k=3$ BCCBB | ACCCA | ABABA

Choosing k Bs 1st 5 slots

$\binom{5}{k}$

5 − k Cs in the 1st 5

5 − (5 − k) = k Cs in these 5

$\binom{5}{k}$

5 − k As here

5 − (5 − k) = k

Select k slots As out these 5

$\binom{5}{k}$

$$\sum_{k=0}^{5} \binom{5}{k}^3 = 1^3 + 5^3 + 10^3 + 10^3 + 5^3 + 1^3$$

$$= 1 + 125 + 1000 + 1000 + 125 + 1$$

$$= 2252$$

Odometer $0 \to 1 \to 2 \to 3 \to 5 \to 6 \to 7 \to 8 \to 9$

odom	base 9 digit
0	0
1	1
2	2
3	3
5	4
6	5
7	6
8	7
9	8

0	0	2	0	0	5

2004_9

$= 2 \times 9^3 + 4$

$= 1462$