COT 3100 10/31/24 9am

① Carefully read through ALL typed notes.

Last Counting Lec

Posted Homework 5 → Due 4 days Project 1.

$S = \{1, 2, 3, \ldots 30\}$

a) How many subset of S of size 5?

$\binom{30}{5}$

b) How many of the subsets in (a) have a ~~min~~ minimum element not equal to 5?

$\binom{30}{5}$ − [Subsets w/ min element 5]

5 in set

4 elements left to choose from set $\{6, 7, \ldots, 30\}$

we can do this in $\binom{25}{4}$ ways since the set above has 25 elements

$$\boxed{\binom{30}{5} - \binom{25}{4}}$$

c) How many subsets w/ min element < 5?

$\{5, 6, \ldots 30\}$

$\binom{30}{5}$ − [Set w/ min element ≥ 5]

↓ any set of 5 chosen from $\{5, 6, \ldots 30\}$

$\binom{30}{5} - \binom{26}{5}$

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$$

$2n$ items n RED n BLUE

ways to select n of the $2n$ items

$\hookrightarrow \binom{2n}{n}$ ways (RHS)

Or we count as follows

R	B	# ways
0	n	$\binom{n}{0}\binom{n}{n}$ +
1	$n-1$	$\binom{n}{1}\binom{n}{n-1}$ +
2	$n-2$	$\binom{n}{2}\binom{n}{n-2}$ +
3	$n-3$	$\binom{n}{3}\binom{n}{n-3}$ +
$\vdots$	$\vdots$	+
i	$n-i$	$\binom{n}{i}\binom{n}{n-i}$ +
$n-1$	1	
n	0	

→ choose 0 red out of n

→ choose n blue out of n

$$\sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i}$$

$$= \sum_{i=0}^{n} \binom{n}{i}\binom{n}{i}$$

$$= \sum_{i=0}^{n} \binom{n}{i}^2$$

COMPLICATED

Consonants when read left to right must be in alphe order

Same for the vowels

AEIO

CCDLMPT

C A C O L E I M O P T

C A C D E I L O M P T

Choose 4 slot out of 11 for vowels

$\binom{11}{4} \times 1 \times 1$

vowels w/ set / set

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3 Oak, 4 Maple, 5 Birch } row perm no two birch trees next to each other

B O _ O _ O B M B M B M _ M B

Can put either 0 or 1 B in each gap btw Os, Ms. Choose 5 slots out of 8 to place birches $\binom{8}{5}$

3
56
35
280
168
1960

orders of Oaks + Maples = $\binom{7}{3}$ choose 3 slots Oaks out of 7

7!
3!4!

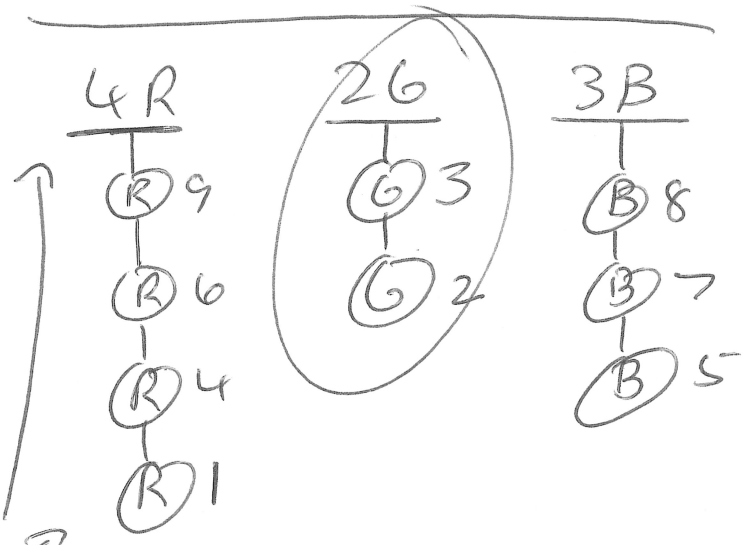
$\binom{8}{5} \times \binom{7}{3} = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 56 \times 35 = 1960$

$$\binom{8}{5} = \binom{8}{3} = \frac{8!}{3!5!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{(1\times2\times3)(5\times4\times3\times2\times1)}$$

$$\binom{7}{3} = \frac{7!}{3!4!}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Target Shooting



Shoot all 9 targets. Want to shoot the lowest one on a string not yet shot. How many ~~ways~~ orders can we shoot down all 9 targets

$\binom{9}{4}$ Choose 4 slots out of 9 to shoot Reds

$\binom{5}{2}$ Chose 2 slots out of 5 left

$\binom{3}{3}$

$$\binom{9}{4}\times\binom{5}{2} = \frac{9!}{4!\,5!}\cdot\frac{5!}{2!\,3!}$$

$\frac{9!}{4!2!3!}$

One to one correspondence btw all ways of shooting the targets and strings of 4Rs 2Gs 3Bs. RGGRBRBBR then that corresponds to this order.

Spider has 8 legs. Hes to put on its socks and shoes. The sock for each leg must go on before the shoe for that leg. But it's possible that shoe for Leg_2 is put on before the sock for Leg_1.

16 objects to put on. In how many orders can it place these objects on its legs adhering to the rules?

A_1 sock 1 B_1 shoe 1, etc.

$A_5 \; A_2 \; B_2 \; A_1 \; B_1 \; A_6 \; B_5 \; A_7 \; B_7 \; A_8 \; B_8 \; A_3 \; A_4$

$B_4 \; B_6 \; B_3$

5 2 2 1 1 6 5 7 7 8 8 3 4 4 6 3

$$\frac{16!}{(2!)^8} = \frac{16!}{2^8}$$

A B

5th vs 4th —L→ 5th B

↓W

vs 3 C —L→ 4th C

↓W

vs 2 D —L→ 3rd A

D

↓W E

1st ←W— vs 1 —L→ 2nd E

D, E, A, C, B

2×2×2×2 = 16 possible outcomes of the 4 matches

LLWL

$A + B + C + D = 98$

A, B, C, D pos odd int

how many sols to this eqn w/ those restrictions?

let $A = 2a+1$, a is a non-neg int
$B = 2b+1$ b,
$C = 2c+1$ c,
$D = 2d+1$ d

$(2a+1)+(2b+1)+(2c+1)+(2d+1) = 98$

$2(a+b+c+d) = 94$

$a+b+c+d = 47$ } combo w/ repetition
$n = 47$
$r = 4$

a, b, c, d non-neg.

$\binom{47+4-1}{4-1} = \binom{50}{3}$

How many permutations of 5 As, 5 Bs + 5 Cs such that no As are in the 1st 5 letters, no Bs in the second 5 letters, no Cs in the last five letters?

$k=3$ BCCBB | ACCCA | ABABA

k Bs in 1st 5

note $5-k$ Cs are placed

$\binom{5}{k}$

Choose $5-(5-k) = k$ slots Cs

$\binom{5}{k}$

we'll have $5-k$ Bs $\Rightarrow$ must place k As

$\binom{5}{k}$ ways

$$Ans = \sum_{k=0}^{5} \binom{5}{k}^3 = 1^3 + 5^3 + 10^3 + 10^3 + 5^3 + 1^3$$

$$= 1 + 125 + 1000 + 1000 + 125 + 1$$

$$= \boxed{2252}$$

Odometer $0 \to 1 \to 2 \to 3 \to 5 \to 6 \to 7 \to 8 \to 9$

0	0	2	0	0	5

2004_9

$= 2 \times 9^3 + 4$

$= 1462$

$0 \to 0$
$1 \to 1$
$2 \to 2$
$3 \to 3$
$5 \to 4$
$6 \to 5$
$7 \to 6$
$8 \to 7$
$9 \to 8$

Digits

2, 6, 8, 9
↓ ↓ ↓ ↓
0 1 2 3

96228
↓ ↓ ↓ ↓ ↓
31024