COT 3100 10/29/24 10^{30} am

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(1) Post an update about course grades tonight.
- POST FORMULA (plug raw numbers not percentages)

A line, B line, C line

(2) Sample Project
- So you can see my expectations

C / Python
↑ ↑
9 10^{30}

(3) Combinations w/ Repetition

How many permutations are there of the letters in ABRACADABRA?

5 As, 2 Bs, 2 Rs, 1 C, 1 D $\longrightarrow$ $\frac{11!}{5!2!2!1!1!}$

another way

11 slots choose 5 A
6 slots choose 2 B
4 slots choose 2 Rs
$\binom{11}{5} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{1} \times \binom{1}{1}$

B A A C R A B D A R A

2 slots choose 1 C
2 slots choose 1 D
$\Longrightarrow$ for C only

Program

Write a program reads in strings of uppercase letters, and for each calculate # perms of letters MOD 10^9+7.

Input (Stdin)

```
n // int n, # cases
ABRACADABRA
MISSISSIPPI
DOCCAM
  :
```

Output

1 int per line ans to each problem

$$\binom{11}{5}\binom{6}{2}\binom{4}{2}\binom{2}{1}\binom{1}{1}$$

cur [11] 6

f [5]

$\binom{cur}{f}$, cur -= f

Combinations w/ Repetition

Taco Bell : tacos
chicken tacos
nachos
burrito
7-layer burrito
mexi pizza
20 items total

$t + c + n + b + s + m = 20$
count the # of non-neg int sols to this eqn.

$(15, 1, 1, 1, 1, 1)$

$\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast|\ast|\ast|\ast|\ast|\ast$

$\ast\ast\ast||\ast\ast|\ast\ast\ast\ast|\ast|\ast\ast\ast\ast\ast\ast\ast\ast\ast\ast$

3 0 2 4 1 10
t c n b s m

n items total
r unique options → r-1 bars

$\frac{25!}{20!5!} = \binom{25}{5} = \binom{25}{20}$

$$\frac{(n+r-1)!}{n!\,(r-1)!} = \binom{n+r-1}{n} = \binom{n+r-1}{r-1}$$

pens
n bars
r-1 stars

choose n out of n+r-1 for stars

↳ choose r-1 slots for bar

a) Must have at least 3 mexican pizzas

$t + c + n + b + s + m = 20$

buy 3 mex pizzas $\Rightarrow$

$t + c + n + b + s + m = 17$ $\quad n=17$ $r=6$

$$\binom{17+5}{5} = \binom{17+5}{17} = \frac{22!}{17!5!}$$

b) Store only has 5 mexican pizzas left $\therefore$

→ ALL − combos w/6 or more

→ $\binom{25}{5} - \binom{19}{5}$ buy 6 mex p $\quad t + c + n + b + s + m = 14$ $\quad n=14$ $r=6$

Alternative Slower route

$$\binom{24}{4} + \binom{23}{4} + \binom{22}{4} + \binom{21}{4} + \binom{20}{4}$$

$m=0 \quad m=1 \quad m=2 \quad m=3 \quad m=4$

hockey stick identity

c) $m \leq 5$ and $t \leq 3$

$t + c + n + b + s + m = 20$

$\binom{25}{5} - \binom{19}{5} - \binom{21}{5} + \underline{\binom{15}{5}}$

All — $m \geq 6$ — $t \geq 4$ — $m \geq 6$ and $t \geq 4$

$t' + c + n + b + s + m' = 10$ $\quad n=10$ $r=6$

$$\binom{25}{5} - \left[\binom{19}{5} + \binom{21}{5} - \binom{15}{5} \right]$$

all combos w $m \geq 6$ OR $t \geq 4$

d) $(3, 1, 2, 0, 1, 2)$

$t + c + n + b + s + m \leq 20$

all in stock!

sols

slack variable

leftover

$= \#$ sols $\; t + c + n + b + s + m + L = 20$ $\quad n=20$ $r=7$

$(3, 1, 2, 0, 1, 2, 11)$ $\quad \boxed{\binom{26}{6}}$

$(0, 2, 5, 12, 0, 0, 1) \leftrightarrow (0, 2, 5, 12, 0, 0)$