

COT 3100 10/29/24 9am

① Sample Project

② Combinations w/ Repetition

Proj 1 (Counting)

Hw 5 (Count/Prob)

Proj 2 (Prob)

↓
Hw 6 (Func/Rel)

Question

How many permutations are there in for arranging the letters in ABRACADABRA.

5 As
2 Bs 1 D
1 C
2 Rs

$$\frac{11!}{5! 2! 1! 2! 1!}$$

Alternative ans

R x x B x R C x B x -

1) Choose 5 slots out of 11 for As

$$\binom{11}{5} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{1} \times \binom{1}{1} \rightarrow X$$

2) Choose 2 slots 6 remaining

3) Choose 2 " 4 remaining

4) Choose 1 " 2 remaining

5) Choose 1 " 1 remaining

BS
RS
C
D
} independent
multiply

Program (input = stdin)

n // # test cases

ABRACADABRA

MISSISSIPPI

DOCCAM

etc.

| uppercase string

≤ 100 chars

per line

Output: # permutations mod 10^9+7 . (output)

Side note: Fermat's Thm $a^{p-1} \equiv 1 \pmod{p}$
if $\gcd(a, p) = 1$, $p \in \text{Prime}$

$$\underbrace{a^{p-2}}_{a^{-1}} \times a \equiv 1 \pmod{p}$$

$$a^{-1} \equiv a^{p-2} \pmod{p}$$

Coding Process

ll : 5, 2, 2, 1, 1

$$\binom{11}{5} \times \binom{6}{2} \times \binom{4}{2} \times \binom{2}{1} \times \binom{1}{1}$$

LEFT 11 6 4 2 1

CUR 8 2 2 + 1

Combinations w/ Repetition

Taco Bell = tacos

chicken tacos

nachos

burrito

7-layer burrito

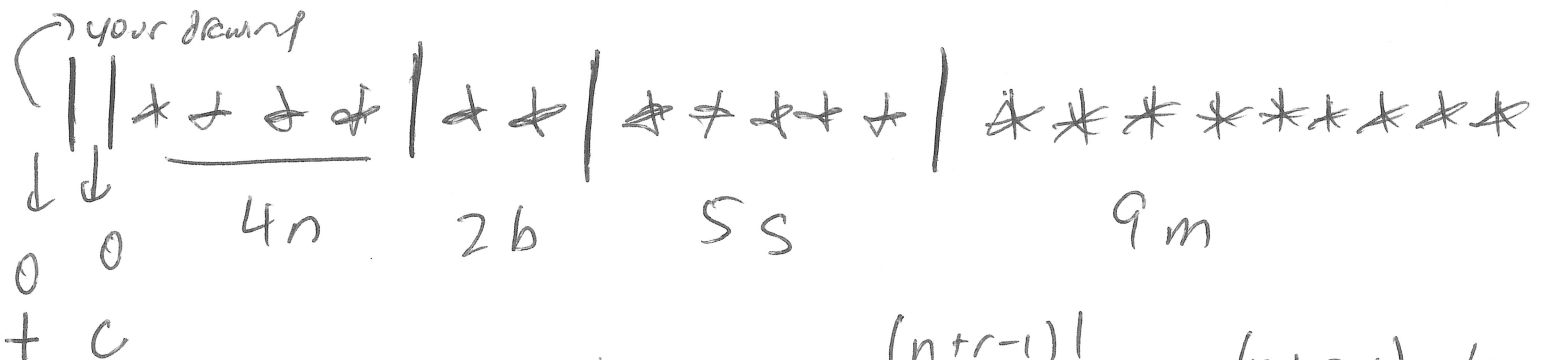
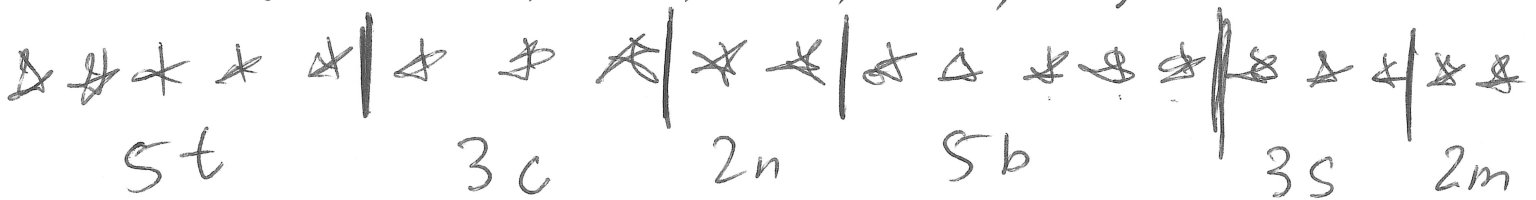
mex pizza

20 items / 6 unique items to choose from
 $n = 20$ $r = 6$

$$t + c + n + b + s + m = 20$$

non-neg int solutions to this eqn $\uparrow$

(5, 3, 2, 5, 3, 2)



n stars
 $r-1$ bars

$$\frac{(n+r-1)!}{n!(r-1)!} = \binom{n+r-1}{n} = \binom{n+r-1}{r-1}$$

$\downarrow$ perm $\downarrow$ choose n slots stars $\downarrow$ choose $r-1$ slots bars

$$t + c + n + b + s + m = 20$$

a) Must have at least 3 mexican pizzas!

buy 3 mexican pizzas, 17 items left to buy

$$t + c + n + b + s + m = 17$$

$$\rightarrow \binom{17+6-1}{6-1} = \binom{22}{5} \text{ ways.}$$

b) Store only has 5 mexican pizzas left :-

i) Separate out $m=0, m=1, m=2, m=3, m=4$ and add ans for each case

$$\binom{24}{4} + \binom{23}{4} + \binom{22}{4} + \binom{21}{4} + \binom{20}{4}$$

ii) Debug this

1st choosing 15 out of 5 (no m)
5 out of 6 (allow m)

this overcounts

5 5 5 5 0 0

(5 5 5 0 0 0), (0 0 0 5 0 0)

(0 5 5 5 0 0), (5 0 0 0 0 0)

iii) ALL - CAN'T DO

one w/ 6 or more mex pizzas

$$\binom{25}{5} - \binom{19}{5}$$

$$t + c + n + b + s + m = 14$$

c) $m \leq 5$ and $t \leq 3$

$$t + c + n + b + s + m = 20$$

$$\binom{25}{5} - \binom{19}{5} - \binom{21}{5} + \binom{15}{5}$$

ALL

~~b~~
 $m \geq 6$

too many
 m

$t \geq 4$

too many
 t



combos
w/ $m \geq 6$
and $t \geq 4$

$$\binom{25}{5} - \left[\binom{19}{5} + \binom{21}{5} - \binom{15}{5} \right]$$

ALL

too
many
 m



too many
 m OR t .

I/E