

COT 3100 Recitation #11: Probability 1
Quiz and Solutions

11/9/2020-11/13/2020

1) Three standard fair six-sided dice are rolled. What is the probability that the total number of dots showing on three faces facing up is 5?

Solution

The sample space is $6 \times 6 \times 6 = 216$ ordered triplets of rolls. Of these, the following triplets add up to 5: (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), and (2, 2, 1). It follows that the desired probability is $\frac{6}{216} = \frac{1}{36}$.

2) In a particular video game, Aliyah's chance of getting a booster pack is 16%. If she gets the booster pack, her chance of completing the mission is 90%. If she does not receive the booster pack, her chance of completing the mission is 30%. In a particular game, Aliyah completed her mission. What is the probability she got a booster pack while completing that mission?

Solution

The probability Aliyah gets the pack and completes the mission is $.16 \times .9 = .144$.

The probability Aliyah doesn't get the pack and completes the mission is $.84 \times .3 = .252$.

The probability Aliyah completes the mission is $.144 + .252 = .396$.

It follows that the probability she got the booster pack given that she completed the mission is

$$\frac{.144}{.396} = \frac{4}{11}.$$

3) If A and B are events and $p(A \cap B) = 1/8$, $p(A | B) = 1/2$. What is $p(B)$?

Solution

$$p(A | B) = 1/2 = p(A \cap B)/p(B) = (1/8)/p(B)$$

$$\text{So, } p(B) = (1/8)/(1/2) = \mathbf{1/4}.$$

4) Ethan is taking a matching quiz with 5 items on it. Unfortunately, Ethan didn't study and he completely guesses the answers, knowing that each of the five words will match to a different definition of the five given. What is the chance that he gets precisely 3 of the definitions correct? (We assume he matches each word to a unique definition listed.)

Solution

There are $5! = 120$ possible answers to the quiz. Denote the answers as permutations of 1, 2, 3, 4 and 5, where the number of correct answers are the number of items x that are in position x . For example, the permutation 5, 2, 3, 4, 1 is a set of possible answers with exactly 3 matching. Notice that to make 3 answers correct, you must simply choose 2 questions to swap answers on. We can choose 2 questions (to swap answers on) out of 5 in 10 ways. Thus, the desired probability is $10/120 = 1/12$.

5) George is buying 10 items from the sweet shop from the following choices: truffle, candied apple, cupcake, brownie, cookie and macaroon. Each combination of items is equally likely. What is the probability that he gets at least one of each item?

Solution

The sample space is the number of combinations with repetition with $n = 10$ and $r = 6$, which is $\binom{10 + 6 - 1}{6 - 1} = \binom{15}{5}$.

Of those combinations, the ones with at least one of each type of sweet would be the number of combinations with repetition with $n = 4$ and $r = 6$, since we must pre-buy 1 of each sweet. This can be done in $\binom{4 + 6 - 1}{6 - 1} = \binom{9}{5}$.

We reduce as a fraction as follows: $\frac{\binom{9}{5}}{\binom{15}{5}} = \frac{9!5!10!}{15!4!5!} = \frac{9!10!}{15!4!} = \left(\frac{9!}{4!}\right)\left(\frac{10!}{15!}\right) = \frac{9 \times 8 \times 7 \times 6 \times 5}{15 \times 14 \times 13 \times 12 \times 11}$

We can cancel 7×6 on the top with 14 on the bottom, yielding 3 on top. Then take that 3 and pair with the 5 on top and cancel the 15. Finally cancel the 9×8 on the top with 12 on the bottom, leaving 6 on the top so our final fraction is $\frac{6}{143}$.