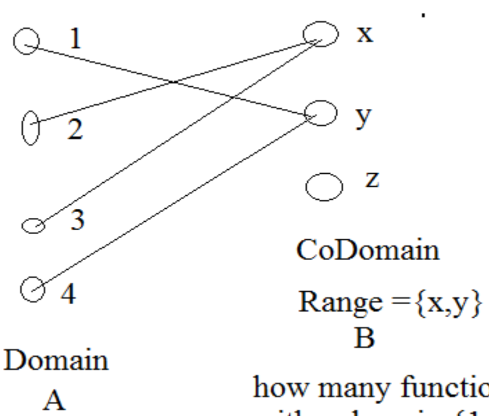
Supplemental Notes to 12/1/2020 Lecture COT 3100

Counting the number of functions with a finite domain and codomain:

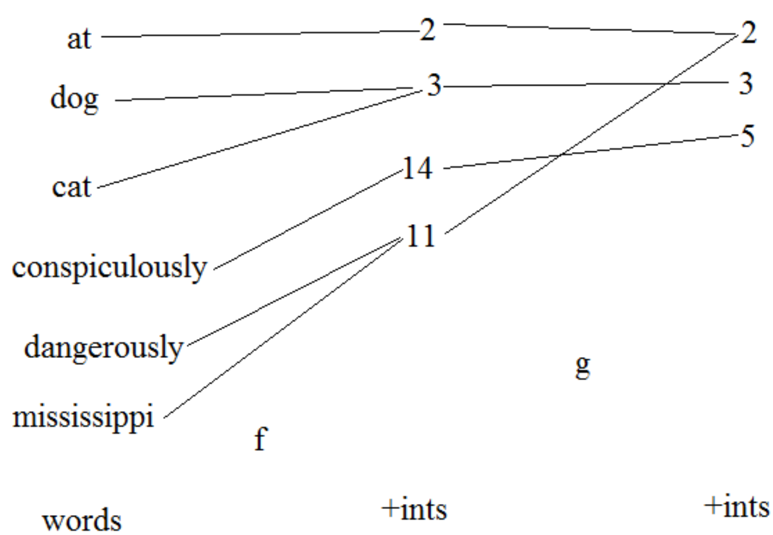


f(1)=x or y or z
f(2)= 3 choices again...

In general # of functions from set A to set B where |A| and |B| are finite is |B| raised to the |A| power.

how many functions can we create with a domain {1,2,3,4} and a codomain {x,y,z}

Function Composition Picture:



Picture of g(f(w))

Can't do the other way around because you can't apply g to a word.

Example of function composition

f(x) = $(3x - 2)^2$ g(x) = e^{5x-7}

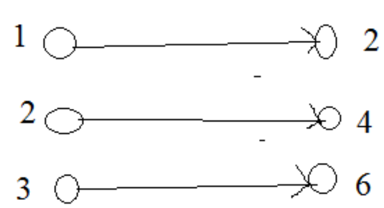
g(f(x)) = g($(3x - 2)^2$) = $e^{5(3x-2)^2-7}$

f(g(x)) = f(e^{5x-7}) = $(3e^{5x-7} - 2)^2$

In most instances changing the order of function composition changes the result. In addition, like the word example, just because g(f(x)) exists doesn't mean that f(g(x)) exists…

Injection, Surjection,Bijection

For an injection, no item in the codomain can have 2 incoming lines



f(x) = 2x

This is an injection...if f(x) = 2x and f(x) = 4, we know x had to be 2.

This is also a surjection if the domain/codomain are reals.

For a surjection, each item in the codomain must have a line coming into it.

Our word function was not surjective...

Bijection is both injective and surjective

For all bijective functions on finite sets, the cardinality of the domain and codomain have to be the same

Let A be the domain, B be the codomain, function f: A ->B

If f is injective |A| <= |B|

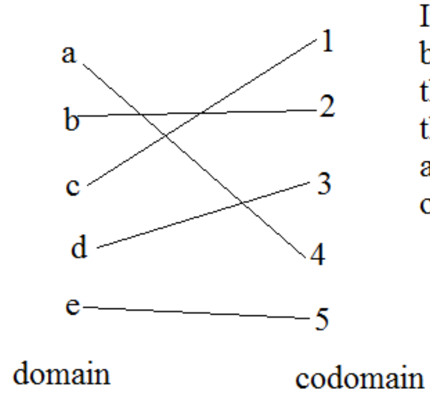
If f is surjective |A| >= |B|

If f is a bijection |A| = |B|

We can only define inverse functions on bijections. In general $f^{-1}(b) = a$ iff $f(a) = b$, it just asks, what input created a specific output.

<u>Bijection btw finite sets viewed as a permutation</u>

Picture of a bijection



In some sense, we can view a bijection between two finite sets as a permutation of the elements of the codomain. In this picture the permutation is (4, 2, 1, 3, 5). Thus, there are n! bijections we can define between 2 sets of size n.

Inverse function example:

Let f(x) = $4x^2 - 16x + 7$ with the domain $x \in (-\infty, 2]$, what is $f^{-1}(x)$? What is the domain and range of $f^{-1}(x)$?

$x = 4y^2 - 16y + 7$

$x = 4(y^2 - 4y + 4) + 7 - 16$

$x = 4(y-2)^2 - 9$

$x + 9 = 4(y-2)^2$

$(y-2)^2 = \frac{x+9}{4}$

$y - 2 = -\sqrt{\frac{x+9}{4}}$, because range of this function must be

$(-\infty, 2]$

$$f^{-1}(x) = 2 - \sqrt{\frac{x+9}{4}}$$

What is the domain of this function? $[-9, \infty)$

$y = \frac{3x-1}{x-1}$, domain all reals except x = 1 Find the inverse function and the domain and range of the inverse function.

Divide the bottom into the top and you get:

Intermediate steps: $y = \frac{3x-3+2}{x-1}$

$$y = \frac{3(x-1)+2}{x-1}$$

$$y = 3 + \frac{2}{x-1}$$

Now to find the inverse function, switch x and y:

$$x = 3 + \frac{2}{y-1}$$

$$x - 3 = \frac{2}{y-1}$$

$$y - 1 = \frac{2}{x-3}$$

$$f^{-1}(x) = 1 + \frac{2}{x-3}$$

Domain of f^{-1}: all real x except x = 3.

Relationship between f(x), f(-x)

When you have $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

There is a relationship between f(x) and f(-x):

$f(-x) = ax^4 - bx^3 + cx^2 - dx + e$, some same terms, but the odd powered terms are flipped.

Always worth plugging in x = 1, -1, 0

$f(0) = e$

$f(1) = a + b + c + d + e$

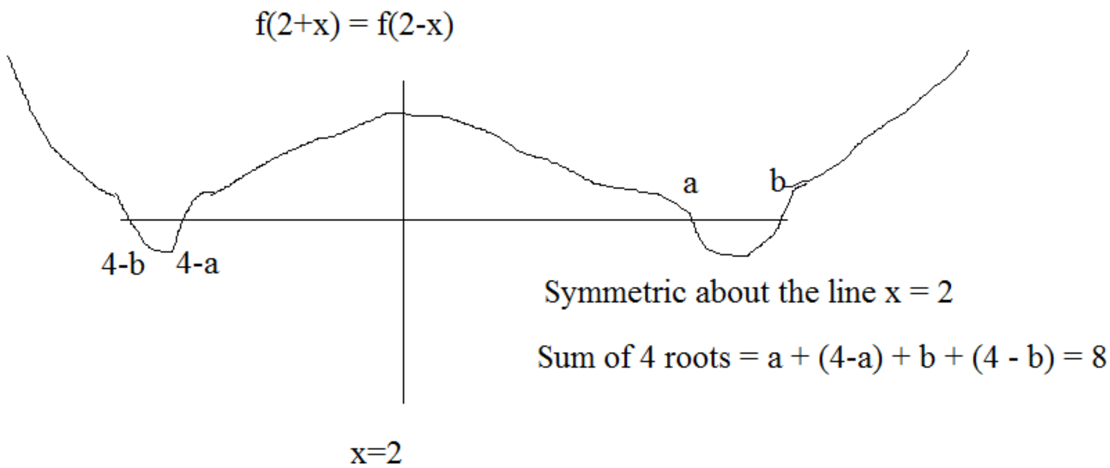
$f(-1) = a - b + c - d + e$

$f(1)+f(-1) = 2a + 2c + 2e$

$f(1)-f(-1) = 2b + 2d$

Really important technique: If you have two polynomials which you know are equal, then you can equate all coefficients.

Symmetric Function Picture



Synthetic Division

$F(x) = 2x^3 - 3x^2 + 5x + 7$

$F(4) = 2(4^3) - 3(4^2) + 5(4) + 7 = 128 - 48 + 20 + 7 = 107$

Divide by x - 4:

	2	-3	5	7
4	2	5	25	107 (f(4) = 107)

A b c d

r a ar+b ar^2+br+c ar^3+br^2+cr+d, by definition f(r).

In CS2 I used to teach this as Horner's Method for polynomial evaluation…notice that we never called the pow function…

$P_2(x) = P_1(x-2) = P_0(x-3)$

$P_3(x) = P_2(x-3) = P_1(x-5) = P_0(x-6)$

$P_{20}(x) = P_0(x-1-2-3-\ldots 20) = P_0(x-210)$