Relations

Sunday, November 22, 2020 12:14 PM

$f(x) = x^2 - 3x$

function

Input	Output
0	0
1	-2
2	-2
3	0
4	4
5	10 ...

Domain (all reals)

Range

all reals $\geq -\frac{9}{4}$

Function is a specific type of relation. In a relation, we'll have two sets (Domain and the Co-Domain for functions…) and we will simply define some subset of the Cartesian Product between the two sets.

$A = \{a, b, c\} \quad B = \{1, 2\}$

$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

A relation over A x B would be any subset of A x B. Thus, there are a total of 2^6 possible relations over A x B. Here is one of them:

$R = \{(a,1), (a,2), (c,2)\}$

A function is a relation where each item in set A maps to exactly one item in set B. But in a regular relation, items in set a are allowed to map to more than one item in set B, like a above, or there are allowed to map to nothing, like b above.

A = set of UCF students (70000+)
B = set of UCF classes (2000+)

R = {(a,b) | student a is taking course b}

(Elise Simms, COT 3100) is an element of R

Pretty much all databases are built on relations and if you are creating a database software, it's extremely important to understand the mathematics behind relations.

Binary Relations over A x A

It's possible for a relation to be neither reflexive, NOR irreflexive.

A = { 1, 2, 3}
R = { (1, 1), (1, 3), (2, 1), (3, 2) }

This is NOT reflexive because it does NOT contain (2, 2).
This is NOT irreflexive because it DOES contain (1, 1).
(Not reflexive does NOT necessarily mean irreflexive.)

aRb is the same as $(a, b) \in R$

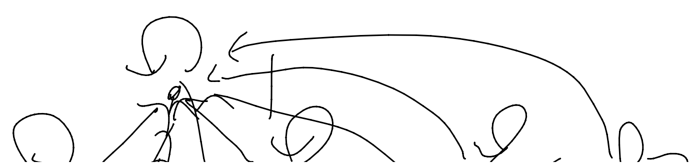
Symmetric means for all a and b in A, if (a, b) is in R, then (b, a) must also be in R.

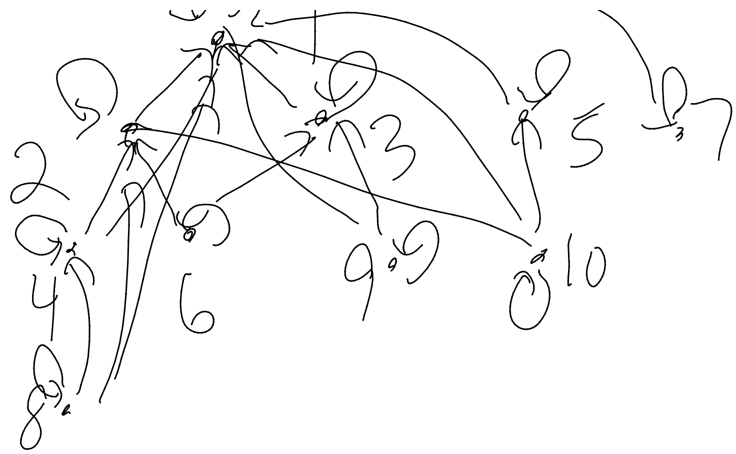
Our set above is NOT symmetric because it contains (1, 3) but does NOT contain (3, 1).

This one is anti-symmetric, in all cases where (a, b) is in R and (b, a) is in R, a = b. (The only satisfying case above is a = b = 1.) (Intuitively, anti-symmetric means that all ordered pairs of unequal elements do NOT have their flip in the relation.)

This one isn't transitive because (1, 3) is in R, (3, 2) is in R but (1, 2) is NOT in R.

Any relation that is reflexive, anti-symmetric, transitive is a partial ordering relation.

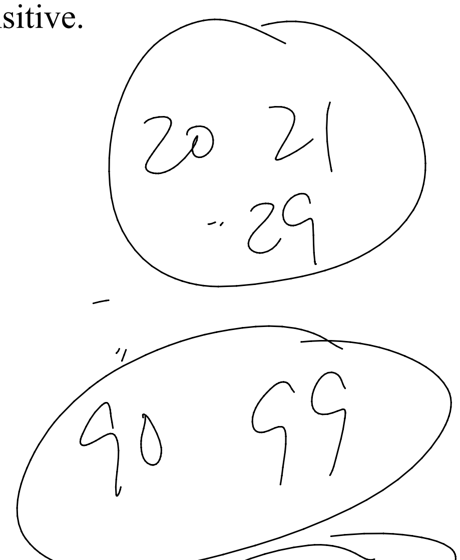




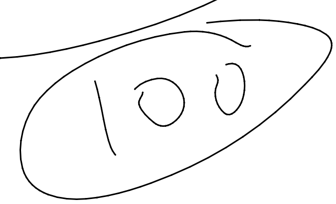
The underlying graph structure of a partial ordering relationship, has one way paths in it that lead to certain end points. For this graph, only 1 is a "terminal" endpoint. Any valid path takes you through proper divisors, for example, 12 --> 6 --> 2 --> 1. It's called a partial ordering because some pairs of items are strictly ordered, like 6 and 3, but not others, like 2 and 3.

Equivalence Relations are reflexive, symmetric and transitive.

Picture of graph of the grading equivalence relation:



In a graph of an equivalence relation, the vertices (which are the values in the set A), are split into different groups. In each group, all pairs of possible connections are present. And, there is NO connections between items in different groups.

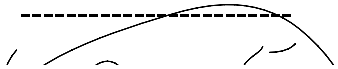


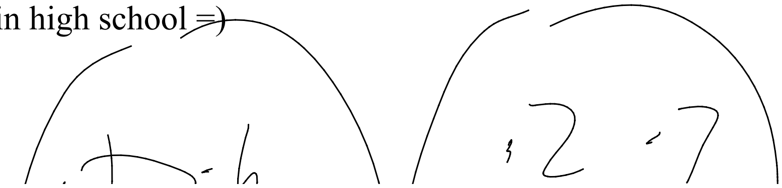
We call each group an "equivalence class". Specifically [x] is the equivalence class of the element x. In this example,

[13] = {10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

And in graph theory, a set of vertices where everything is connected is called a clique…just like groups in high school =)

Mod Graph (mod 5)





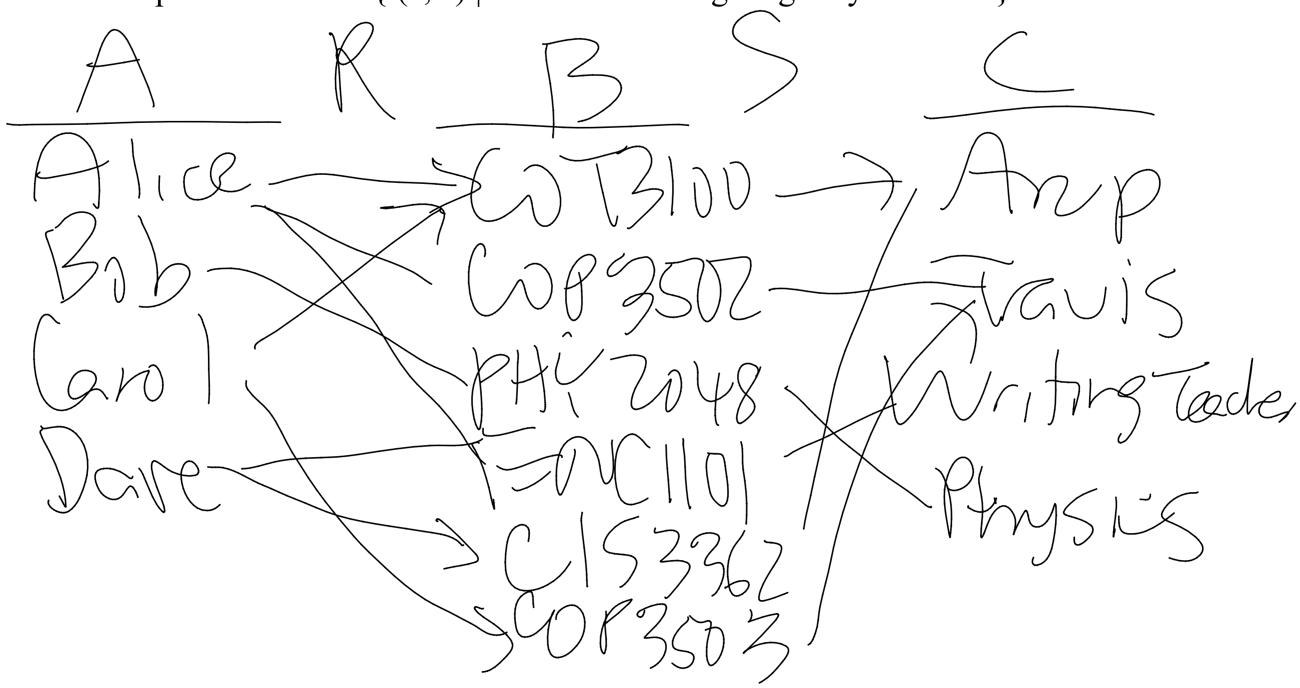
Mod Graph (mod 5)

All the numbers in each circle are equivalent to each other under mod 5…since all are connect.

A equivalence relation partitions a set A into equivalence classes.

Practical Example or Relation composition:

A = set of UCF students
B = set of UCF classes
C = set of UCF teachers.
R = {(a, b) | student a is in class b}
S = {(b, c) | class b is being taught by teacher c}
S composed with R = { (a, c) | student a is being taught by teacher c}



Try to prove S comp R intersect T comp R subset of (S inter T) comp R (a,c) element of S comp R intersect T comp R

1. (a, c) element of S comp R And
2. (a, c) element of T comp R

If #1 is true, by definition there is some b such that (a, b) is in R, (b, c) is in S.

If #2 is true, by definition, there is some d such that (a, d) is in R and (d, c) is in T.

Problem here is that I have no idea of any common element between S and T. For all we know there is NO intersection!!!

For a relation if

R = { (a, b) | some condition}

R^{-1} = { (b, a) | some condition}

R = { (1, 1), (1, 3), (2, 3), (3, 2) }
R^{-1} = { (1, 1), (3, 1), (3, 2), (2, 3) }