Probability Notes – 11/17/2020
Agenda

1. Binomial Distribution
2. Continuous Random Variables
3. Sample Problems

Binomial Distribution

Have some trial and you have a probability of success p.

If we repeat n trials that are all independent, what is the probability of exactly k success?

N = 5, k = 3, p = .3

Success, Success, Failure, Success, Failure

.3 x .3 x .7 x .3 x .7

More generally:

p x p x (1-p) x p x (1-p) …= $p^k(1-p)^{n-k}$, this is the probability of any string of k successes and n-k failures.

Question: How many different strings or orderings are there of k successes and n-k failures?

It's the number of permutations of k S's and n-k F's.

It's the number of ways to choose k slots out of n slots for the successes.

It's the number of ways to choose n-k slots out of n slots for the failures.

Answer = $\binom{n}{k} p^k (1-p)^{n-k}$

Interestingly enough, consider the following expansion:

1 = $((1-p) + p)^n = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$

Couple items about the binomial distribution

E(X) = np (intuitive result)

Var(X) = np(1-p) (Square root of variance is standard deviation)

The binomial distribution is pretty close to the normal distribution (Z-values), so for a binomial distribution you can calculate probabilities like what's the probability that I get more than k success or less than k successes by using just the normal distribution table and the values of Expectation and Variance Above…

Practical Use: Before I took over the foundation exam, there was one exam printed for each student, and it was single sided.

I looked at the historical data and realized that (a) it's binomial distribution, and (b) only about 80% of students historically even showed up, of the ones who signed up! (I knew n and I knew p, so I could use the gaussian curve to calculate that with 98% probability no more than x number of people would show up.)

So I figured out two things: 1) Didn't need print an exam for every student

2) Double side the thing!!!

Continuous Random Variables

In a discrete random variable, your variable takes on different values with different probabilities. But the value of the variable is one of a few distinct possibilities. (It can also be infinite, but even in this setting it's only certain values that have positive probability associated with them.)

In continuous random variable, the probability of getting any particular value is 0, but there are positive probabilities for getting a value within a range.

General Probability Problem Solving Advice

Any time you're asked for, probability that at least one, or at least two… It might be a good idea to calculate the opposite (probability that none are the same…) – **subtraction trick!**

Biased Coin Question – Alternate Approach

For two coins that are fair, we get 1H, 1T 50% of the time. We get a head on this last coin 75% of the time because it's biased.

For two coins that are fair, we get 2H 25%, then we get a Tail on this last coin 25% of the time, because it's biased.

P (HT and ending on a H) = .5 x .75 =3/8

P(2H and ending on a T) = .25 x .25 = 1/16

P(2H and T) = 3/8 + 1/16 = 7/16

If two events are independent, then we have p(A|B) = p(A), intuitively, knowing B occurred doesn't change the chance of A occurring. P(A and B) = p(A)*p(B). So knowing two things are independent really simplifies things!

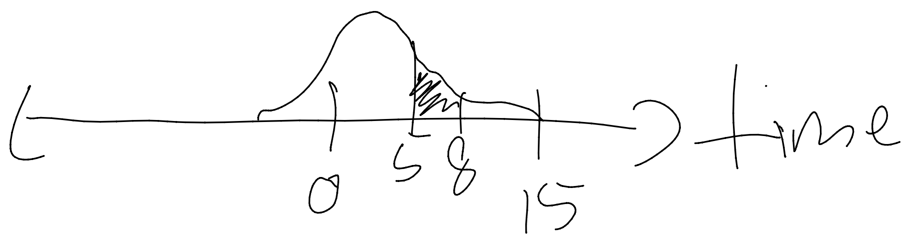
Mutually exclusive means p(A and B) = 0, and p(A or B) = p(A) + p(B).

More Qs

1) Straight binomial
2) 50 people total, 40 tv, 21 comp, 4 own neither, 46 own tv or comp, probability of owning both a tv and computer. Sample space = 50. How many of these people own both?
   |A or B| = |A| + |B| - |A inter B|
   46 = 40 + 21 - |A inter B|
   |A inter B| = 61 – 46 = 15
   Probability = 15/50 = 3/10
   # of people who own computer = 21, # of people who own both = 15
   Conditional prob = 15/21 = 5/7
3) On one note
4) Binomial again, for part b, add up prob of 0 prizes, 1 prize and 2 prizes
5) On one note
6) Skipped – exercise for the reader
7) Skipped – exercise for the reader
8) 1/5 + 2/5 + 1/10 + x = 1, so x = 3/10
   We can get six (2,4), (3,3) or (4,2)
   p = (4/10)(3/10) + (1/10)(1/10) + (3/10)(4/10) = (12+1+12)/100 = 1/4

CRVs - for COT 3100 Lecture (11/17/2020)

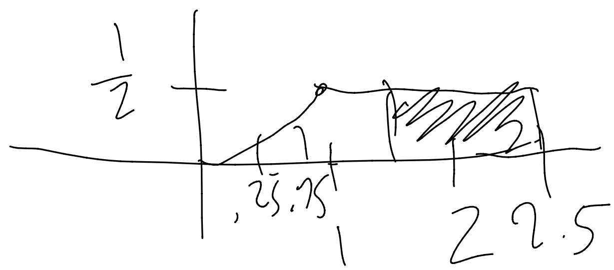
Tuesday, November 17, 2020 1:59 PM



p(5<X<8) = area under curve shown above in between x = 5 and x = 8.

total area under the curve for a CRV = 1

X = x/2, if 0 < x < 1
= 1/2, if 1 < x < 2.5



p(1.5 < X < 2.5) = 1/2 since it's a rectangle with base 1 height 1.2

p(.25 < X < .75) = trapezoid area, more general integral

$$\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_{1/4}^{3/4} = \frac{1}{4}\left(\frac{9}{16}-\frac{1}{16}\right) = \boxed{\frac{1}{8}}$$

$\boxed{8}$

E(X) $= \int_{-\infty}^{\infty} x \cdot p(x)$

$$= \int_0^1 x \cdot \frac{x}{2}\, dx + \int_1^{2.5} x \cdot \frac{1}{2}\, dx$$

$$= \left.\frac{x^3}{6}\right|_0^1 + \left.\frac{x^2}{4}\right|_1^{2.5}$$

$$= \frac{1}{6} + \frac{25}{16} - \frac{4}{16}$$

$$= \frac{1}{6} + \frac{21}{16} = \frac{8+63}{48}$$

$$= \boxed{\frac{71}{48}}$$

Variance/Standard Deviation:

Var = E(X^2) - [E(X)]^2

E(X^2) = $\int^{1} x^2 \cdot \frac{x}{2}\, dx + \int^{2.5} x^2 \frac{1}{2}\, dx$

$$E(X^2) = \int_0^1 x^2 \cdot \frac{x}{2}\,dx + \int_1 x^2 \cdot \frac{1}{2}\,dx$$

Generally,

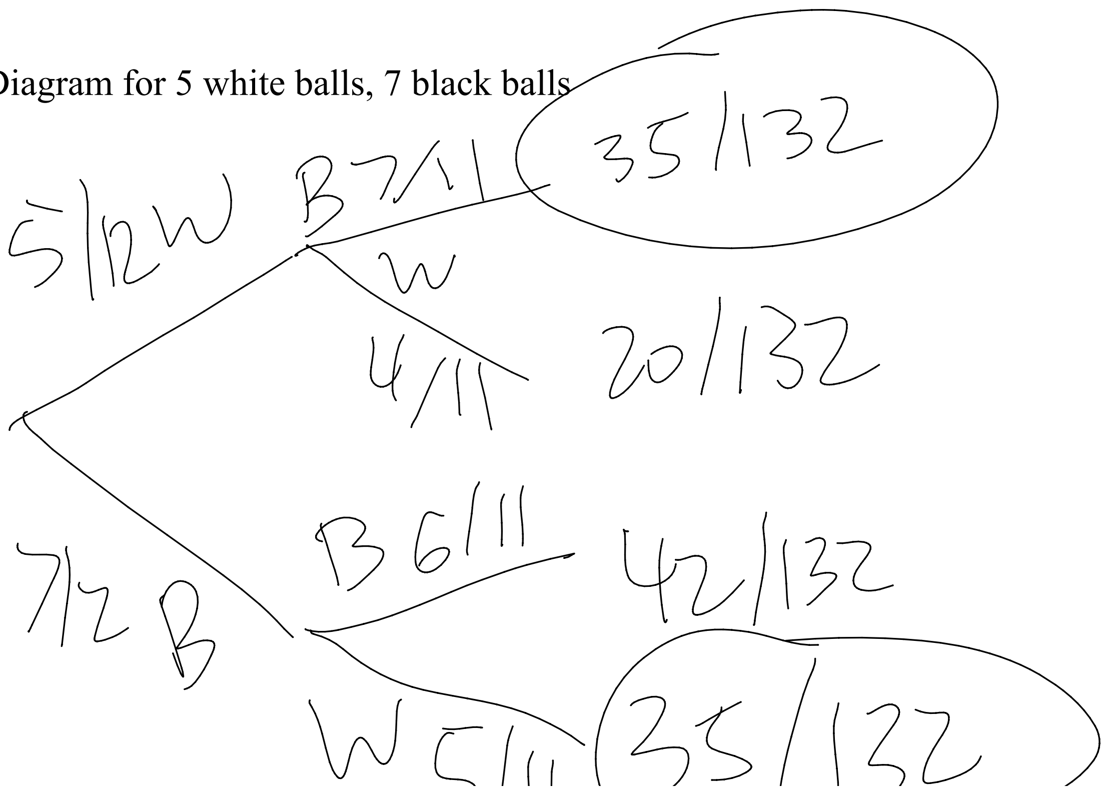
$$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot p(x)$$

median is the value of m such that

$$\int_{-\infty}^{m} p(x) = .5$$

mode is where the maximum of the function is…

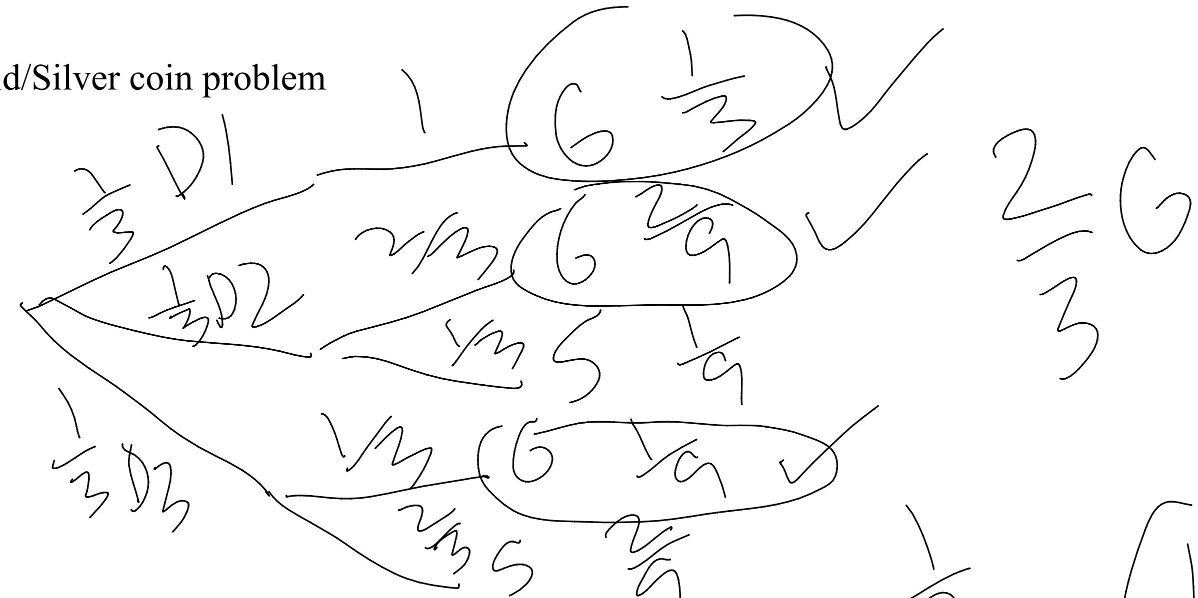
Tree Diagram for 5 white balls, 7 black balls



W 5/11 $35/132$

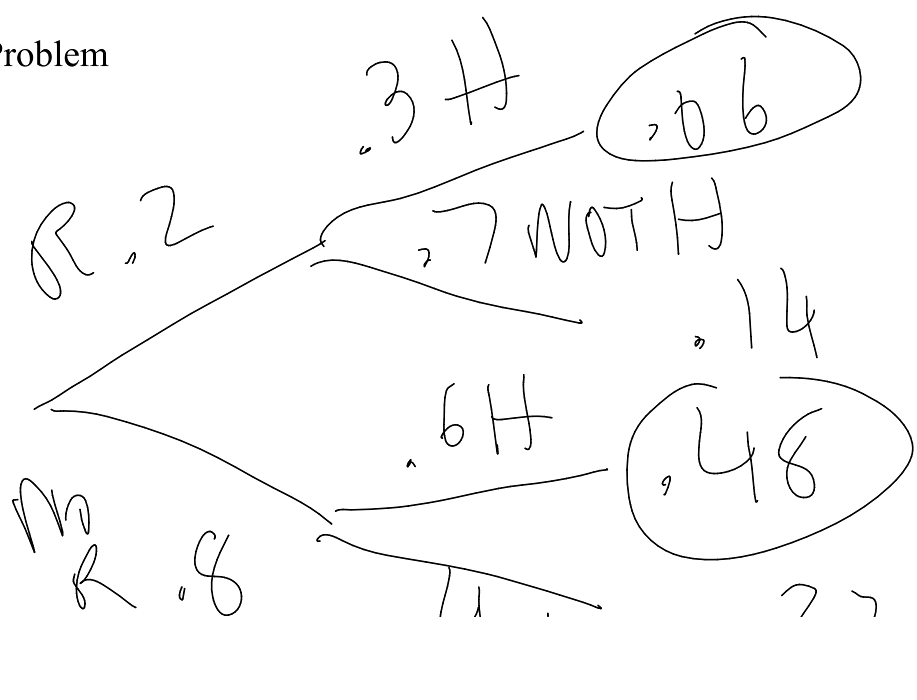
$$\frac{70}{132} = \frac{35}{66}$$

Gold/Silver coin problem



$$P(D3|G) = \frac{1/9}{2/3} = \frac{1}{6}$$

Weather Problem



R .8 .4 Not H .32

p(Hot) = .06 + .48 = 0.54
p(Rain | Hot) = p(Rain and Hot)/p(Hot) = .06/.54 = 1/9

Question 9: DRV infinite sum

$$\sum_{x=0}^{\infty} k\left(\frac{2}{3}\right)^x = 1$$

$$k \sum_{x=0}^{\infty} \left(\frac{2}{3}\right)^x = 1$$

$$k \cdot \frac{1}{1-\frac{2}{3}} = 1$$

$$k \cdot 3 = 1$$

$$k = 1/3$$

What is the expected value of this discrete random variable?

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{1}{3} \cdot \left(\frac{2}{3}\right)^x$$

$$S = 0 + \frac{1}{3}\cdot\frac{2}{3} + 2\cdot\frac{1}{3}\cdot\frac{4}{9} + 3\cdot\frac{1}{3}\cdot\frac{8}{27} + \cdots$$

$$-\frac{2}{3}S = 0 + 1\cdot\frac{1}{3}\cdot\frac{4}{9} + 2\cdot\frac{1}{3}\cdot\frac{8}{27}$$

$$\frac{1}{3}S = 0 + \frac{1}{3}\cdot\frac{2}{3} + \frac{1}{3}\cdot\frac{4}{9} + \frac{1}{3}\cdot\frac{8}{27} +$$

$$\frac{1}{3}S = \frac{1}{3}\left[\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots\right]$$

$$S = \frac{2/3}{1-\frac{2}{3}} = \frac{2/3}{1/3} = \boxed{2}$$

$$X = \begin{cases} 0, & \text{w/ prob } 1/3 \\ 1, & \text{w/ prob } 2/9 \\ 2, & \text{w/ prob } 4/27 \\ 3, & \text{w/ prob } 8/81 \end{cases}$$

$$\downarrow$$