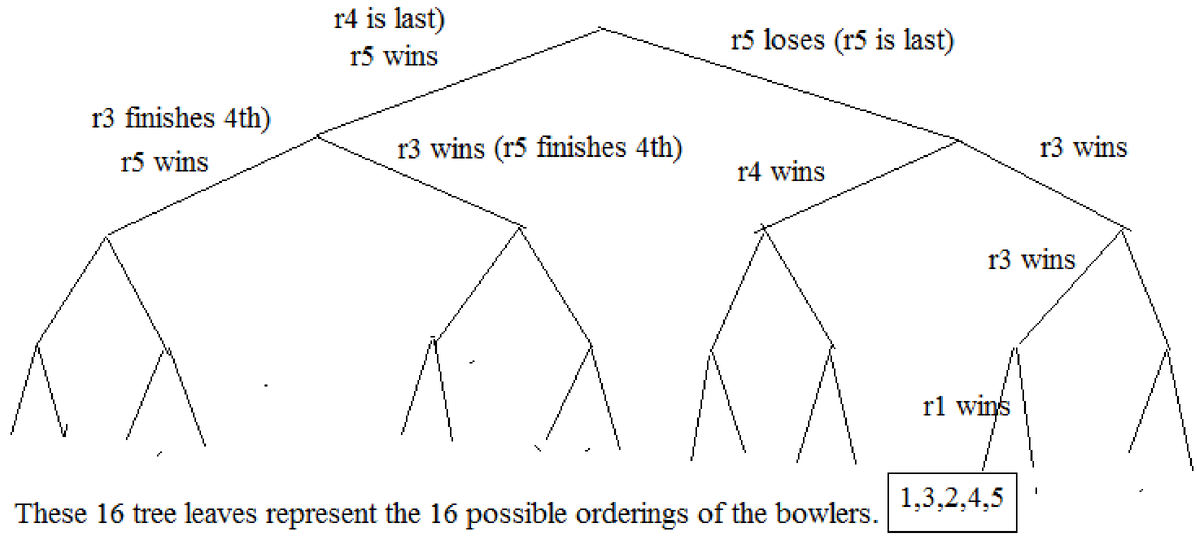
Bowling problem visualization



These 16 tree leaves represent the 16 possible orderings of the bowlers.

One branch, where r5 loses, r4 loses, r2 loses and then r3 loses has been filled in.
In this branch, the final ordering is r1, r3, r2, r4 and r5.

Combos w/ Repetition Alternative Restrictions

a + b + c + d = n

a must be odd, b, c, d can be anything

a = 2a' + 1

2a' + 1 + b + c + d = n

2a' + b + c + d = n-1

Kinda of get stuck…might be a fix, but I can't think of it, so we'll leave this and come back to it, if my other ideas fail.

All my solutions have either a even or a odd…how can I decide the relative # number of these solutions…

Summation way would be to sum over all possible values of a. (Ugly doesn't give us a closed form…)

# of solutions to a + b + c + d = n with a being odd equals the number of solutions to a + b + c + d = n-1 with a being even.

Define g(n, even) = # of solutions where a is even

Define g(n, odd) = # of solutions where a is odd.

g(n, odd) = g(n-1, even)

Define f(n) = g(n, even) + g(n, odd) = g(n, even) + g(n-1, even)

f(n) = $\binom{n+3}{3}$**.**

If I were to write a program, I would solve g(0, even), g(0, odd), etc. the first few base cases cases, and then I'd write a for loop to build up the answer.

g(0, odd) = 0, g(0, even) = 1, f(0) = 1
g(1, odd) = 1, g(1, even) = 3, f(1) = 4
g(2, odd) = 3, g(2, even) = 7, f(2) = 10
g(3, odd) = 7, g(3, even) = 13, f(3) = 20
g(4, odd) = 13, g(4, even) = 22, f(4) = 35

Here is some python code:

```
def combo(n,k):
    res = 1
    for i in range(k):
        res = res*(n-i)
        res = res//(i+1)
    return res

odd = [0]
even = [1]
f = [1]
for i in range(1, n+1):
    odd.append(even[-1])
    f.append(combo(i+3,3))
    even.append(f[-1]-odd[-1])
print("Num solutions with a odd =",odd[-1])
```

Generating Functions Solution

$(1 + x^2 + x^4 + \ldots)(1 + x + \ldots)^3 = \left(\frac{1}{1-x^2}\right)\left(\frac{1}{1-x}\right)^3 = \left(\frac{1}{1+x}\right)\left(\frac{1}{1-x}\right)^4$

$= (1 - x + x^2 - x^3 \ldots)(1 + 4x + 10x^2 + 20x^3 + \ldots)$

The coefficient to x^n in this expansion is

$$\sum_{i=1}^{n+1} \frac{i(i+1)(i+2)}{6}(-1)^{n+1-i}$$

This doesn't look like a really fun sum ☹

If the sum has an even # of terms, it's not so bad, since we can split the sum into pairs of terms and then factor out stuff.

If you look at the table my program prints, you can see that the terms in both columns come from an alternating sum of the items in the total column.

Divisibility Counting Problem

1 2 3 4 5 6 7 8 9 10

Problem: 6 was canceled twice!!! So, every 6th number is double counted…

Fully, we use the Inclusion-Exclusion Principle for 3 sets in this problem, using 2, 3 and 5.