Notes for Combos w/Repetition

Tuesday, November 3, 2020 1:43 PM

There's a one to one correspondence between
1) Arrangements of 8 stars and 4 bars
2) # of non-neg int sols to x1 + x2 + x3 + x4 + x5 = 8

To prove 1-to-1 correspondences, what you need to do is prove that any arbitrary item counted in #1 maps to a unique item counted in #2, and any item counted in #2 maps to a unique item counted in #1.

***|**|*|*|* --> (3, 2, 1, 1, 1) (Bins are different colors)

******||**|| --> (6, 0, 2, 0, 0)

Also, do this the other way:

(2,0,3,3,0) --> **||***|***|

That means the answer to our original question is just the number of ways to arrange these stars and bars, which is 12!/(8!4!) - viewed as permutations, OR (12 C 4) - viewed as a choosing 4 slots out of 12 for the bars, OR (12 C 8) - viewed as choosing 8 slots out of 12 for the stars.

For the inequality problem, given any solution to

x1 + x2 + x3 + x4 + x5 + x6 + x7 <= 30

We can map it to a unique solution to

x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 = 30

(3,0,2,5,1,2,6) --> (3,0,2,5,1,2,6,11), we can calculate x8 uniquely given the other 7 items.

Sample Problem
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x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 = 40
x1 >= 2, x5 >= 3, x7 >= 4, x2 <= 3, x3 <= 10

Step 1: give 2 to x1, 3 to x5 and 4 to x7, so instead of distributing 40, we are now distributing 31.

Now we want # of solutions to

x1' + x2 + x3 + x4 + x5' + x6 + x7' + x8 + x9 = 31,
where x1' = x1 - 2, x5' = x5 - 3, and x7' = x7 - 4,
new restrictions on x1', x5' and x7' are they are non-neg ints.

Now, we must deal with x2 <= 3, x3 <= 10…

First do all solutions n=31, r=9… (31 + 9 - 1) C (9-1) = (39 C 8)

Now, let's count all solutions with x2 > 3.
So let's add 4 to x2 right at the beginning, so we have 31-4 = 27 items left to distribute amongst 9 bins, which we can do in
(27 + 9 - 1) C (9-1) = (35 C 8) ways.

Now, let's count all solutions with x3 > 10.
So let's add 11 to x2 right at the beginning, so we have 31-11 = 20 items left to distribute amongst 9 bins, which we can do in
(20 + 9 - 1) C (9-1) = (28 C 8) ways.

But, in the last two parts, we double subtracted combos with x2 >= 4 and x3 >= 11. Now, we must add these back in. So, let's distribute 4 to x2 and 11 to x3, so we have 31 - 15 = 16 left to distribute amongst 9 variables, which we can do in
(16 + 9 - 1) C (9-1) = (24 C 8) ways.

Final answer = (39 C 8) - (35 C 8) - (28 C 8) + (24 C 8)

Note: I want you to leave your answers like this.

COMBINATORICS Question

All vowels in group

OIAOI - vowel super letter
2s C, M, B, N, T, R, S

9 letters…. 9!/2! ways since there are 2 Cs
BUT, within my super-letter, I can rearrange my vowels in 5!/(2!2!) ways, so the total answer is

(9!/2!)*(5!/(2!2!)) = 9!5!/8

Ice Cream Problem Part B
----
7 scoops all different flavors, 7 unique toppics

T1 --> 10 flavor
T2 --> 9 flavor
T3 --> 8 flavors
T4 --> 7 flavors
T5 --> 6 flavors
T6 --> 5 flavors
T7 --> 4 flavor
10 x 9 x 8 x 7 x 6 x 5 x 4 = (10 P 4) = 10!/3!

Alternate way of looking at it

Choose our scoops in (10 C 7) ways…

Now, whatever I chose for the first scoop, I have 7 choices for its topping, whatever I chose for the second scoop, I have 6 choices for its topping,

(10 C 7) * 7! = 10!/(3!7!)*7! = 10!//3!