**Prove:** If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Goal is to show $A \subseteq C$.

Let x be an arbitrary element of A. We must show that x belongs to C.

Since x belongs to A and $A \subseteq B$, is given, it follows that x belongs to B.

Since x belongs to B and $B \subseteq C$, is given, it follows that x belongs to C, as desired.

300 students in COP 3502 (300 - 120 who are only in COP 3502)

320 students in COT 3100 (320 - 120 who are only in COT 3100)

120 students in both courses this semester

Total = 180 (only cs1) + 200 (only disc) + 120 (both)

= 500 total in either class.

If we were to try to figure out 500 another way, intuitively we would do 300 + 320 = 620, but when we did this, we overcounted the 120 people who are taking both. In particular, we counted them twice, so just subtract them out.

$$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$$

This formula is my first intuitive approach, and you may use this result in future work without stating it's proof. This formula breaks up the counting of a union into three disjoint sets. (Two sets are disjoint if their intersection is empty.)

$$|A \cup B| = |A - B| + |B - A| + |(A \cap B)|$$

Actual principle says:

$$|A \cup B| = |A| + |B|$$

$$-|(A \cap B)|$$

Goal for I/E problem:

Find the following set cardinalities:

$|A \cup B| = |A| + |B| - |A \cap B|$

= 14 + 13 - 5 = 22

$|A \cup C| = |A| + |C| - |A \cap C|$

20 = 14 + |C| - 7

20 = 7 + |C|

|C| = 13

$|B \cap C|$

$|A \cup B \cup C|$=

$|A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

$25 = 14 + 13 + 13 - |B \cap C| - 5 - 7 + 3$

$25 = 40 - 12 + 3 - |B \cap C|$

$25 = 31 - |B \cap C|$

$|B \cap C| = 6$

4. If $A \cap B = \varnothing$, then $B \subseteq \neg A$

Let x be an arbitrary element of B. We aim to show that x belongs to the complent of A.

Since x belongs to B and $A \cap B = \varnothing$, x can NOT belong to A, because if it did, $A \cap B$ would not be empty (it would contain x). It follows that x does not belong to A. But, if x doesn't belong to A, by definition of set complement it belongs to the complement of A, as desired.

Construct counter example

Prove or disprove: $A - (B - C) = (A - B) \cup C$.

This is false.

Consider the following counter example:

$A = \{1\}$ $B = \{2\}$ $C = \{1,2\}$

$B - C = \{\}$

$A - (B - C) = \{1\}$

$(A - B) \cup C = \{1,2\}$

$\{2\} - \{1,2\} = \{\}$ because 1 CAN NOT BE IN THE ANSWER BECAUSE 1 is NOT in the first set.

$(x + y)^2 = x^2 + 2xy + y^2$

$(3x^3 + (2x - 1))^2 = 9x^6 + 2(3x^3)(2x-1) + (2x-1)^2$