Assignment 1 a1 percent of course grade
Assignment 2 a2 percent
Let there be n assignments thus
$a1 + a2 + a3 + \ldots + an = 100$
a1 = p1 points out of q1 points
a2 = p2 points out of q2 points
Average = $((a1)*(p1/q1) + (a2)*(p2/q2) + (ak)(pk/qk))/((a1+a2+\ldots ak)/100)$ as a percentage

A1 2% 18 out of 25
A2 5% 50 out of 50
A3 10% 25 out of 40

$(2*(18/25) + 5(50/50) + 10*(25/40))/ ((2+5+10)/100)$
$= 12.69/.17$
$\sim 74.65\%$

<u>Set stuff</u>

Look at first set table on the second set of set notes…

For a set proof, when proving equality between sets A and B,

We must show two things:

(1) for all x in A, x is in B

(2) for all x in B, x is in A

We can ALSO show (2) by taking the contrapositive and showing that

(3) for all x NOT in A, x is NOT in B

A set table does

(1) and (3)

Consider an arbitrary element x. This element x must satisfy exactly one of these eight categories:

1) not in A, not in B, not in C

2) not in A, not in B, in C

3) not in A, in B, not in C

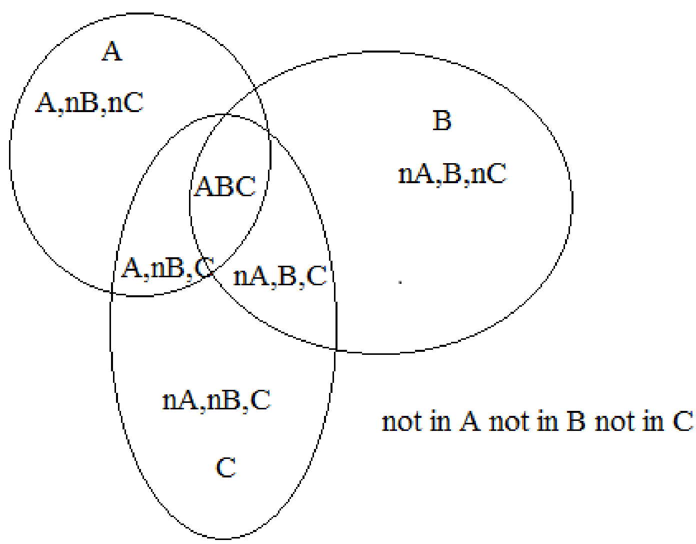
4) not in A, in B, in C

5) in A, not in B, not in C

6) in A, not in B, in C

7) in A, in B, not in C

8) in A, in B, in C



A proof in words would look like:

Consider an arbitrary element that is not in A, not in B and not in C. Then this element is…both elements are NOT in (B inter C) and "Big Set"

So this would occur 8 times for the 8 cases.

Set Proof Example

If A $\cap$ B = $\varnothing$, then B $\subseteq \neg$A.

Goal: Prove B is a subset of the complement of A.

Assumed information: A $\cap$ B = $\varnothing$.

If we want to prove that one set is a subset of another, we will take an arbitrarily chosen item of the first set and attempt to show it belongs to the second set.

Let x be an arbitrarily chosen item from the set B. (Let $x \in B$.)

Since A $\cap$ B = $\varnothing$, it follows that x does NOT belong to A, because if it did, A $\cap$ B ≠ $\varnothing$, because it would contain x.

If x does NOT belong to A ($x \notin A$), then, by the definition of set complement, it follows that $x \in \bar{A}$. Since we've shown that this arbitrarily chosen element belongs to the complement of A, we can conclude that B $\subseteq \neg$A as desired.

The contrapositive is:

If B is NOT a subset of the complement of A, then A intersection B is NOT empty.

Assume that B is NOT a subset of the complement of A. Therefore, there must exist some element x such that x belongs to B and x does NOT belong to the complement of x.

$$\exists x | x \in B \wedge x \notin \bar{A}$$

By definition of set complement, if $x \notin \bar{A}$, $x \in \bar{\bar{A}}$, but by double negation we have that $x \in A$. Thus, we have found an element x such that $x \in B \wedge x \in A$. This means that the intersection of A and B is non-empty as desired.

How would you do this with a set table:

A	B	$\bar{A}$	A $\cap$ B
0	0	1	0
0	1	1	0
1	0	0	0
1	1	0	1

Here is a set table for the interaction between A and B. Since we are given that $A \cap B = \emptyset$, we know that there are NO elements x in the category for the last row.

Thus, the only possible rows that are relevant to the proof are the first three, where the if clause is true:

A	B	$\bar{A}$	$A \cap B$
0	0	1	0
0	1	1	0
1	0	0	0

If we want to show that B is a subset of $\bar{A}$, we must show that for any arbitrary element in B, it also belongs to $\bar{A}$. Among these three cases, in the one case that an arbitrary element belongs to B, it does indeed belong to $\bar{A}$. It follows that if the intersection of A and B is empty, then B is a subset of the complement of A.

Proof by contradiction

If $A \cap B = \emptyset$, then $B \subseteq \neg A$.

We will assume the opposite of what we are trying to prove, that B isn't a subset of the complement of A.

Therefore, there exists an element x that is in B and not in the complement of A. If x isn't in the complement of A it's in A. Therefore x is in both A and B. But this contradicts the given information that the intersection of A and B is empty.

<u>Prove that $(B \subset C) \Rightarrow (B - A) \subset (C - A)$.</u>

Goal: To show that $(B - A) \subseteq (C - A)$.

We use direct proof.

Let x be an arbitrarily chosen element of B - A. Our goal is to prove that x belongs to C - A.

By definition of set different, x belongs to B and x does NOT belong to A. ($x \in B \wedge x \notin A$.)

Since we are given that B is a subset of C and we know that x belongs to B, by definition of subset, we ascertain that x belongs to C. ($x \in C$).

Since $x \in C \wedge x \notin A$, by definition of set difference x$\in C - A$.

Is this following statement true:

If (B – A) $\subseteq$ (C – A), then (B $\subseteq$ C)

This is false.

To prove it's false, consider the following counter-example:

A = {1}, B = {1}, C = {}

B - A = {}, C - A = {}, thus (B – A) $\subseteq$ (C – A), but B is NOT a subset of C because 1 belongs to B but doesn't belong to C.

Is this following statement true:

If (A – B) $\subseteq$ (A – C), then (B $\subseteq$ C)

This is false. Consider this counter example:

A = {1}, B = {1}, C = {}

A - B = {}, A - C = {1}, so it's true that (A – B) $\subseteq$ (A – C), but since B contains 1 and C doesn't contain 1, B is NOT a subset of C.

If (A – B) $\subseteq$ (A – C), is there anything we can guarantee???

<u>Is this true or false?</u>

If A x C $\subseteq$ B x C, then A $\subseteq$ B

Turns out this one is FALSE!

Consider the following counter example:

A = {1}, B = {}, C = {}

A x C = {}, B x C = {}, but A is not a subset of B.

However, the following modified statement is true:

If A x C $\subseteq$ B x C AND C is non-empty, then A $\subseteq$ B.

We must show that for an arbitrarily chosen element x that belongs to A, that x also belongs to B.

Since C is non-empty, there exists some element y that belongs to C.

Therefore, by definition of Cartesian Product, (x, y) belongs to A x C.

By definition of subset and the fact that A x C $\subseteq$ B x C, it follows that (x, y) belongs to B x C.

Now, by definition of Cartesian Product, if (x, y) belongs to B x C, it follows that x belongs to B, as desired.

$$\wp(A) \cup \wp(B) \subseteq \wp(A \cup B)$$

A={1,2}, B={2,3,4},

P(A) = { {}, {1}, {2}, {1,2} }

P(B) = { {}, {2}, {3}, {4}, {2,3}, {2,4}, {3,4}, {2,3,4} }

$$\wp(A) \cup \wp(B) = \{\{\quad\}, \{1\}, \{1,2\}, \{2\}, \{3\}, \{4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{2,3,4\}\}$$

$$\wp(A \cup B) = all\ subsets\ of\ 1,2,3,4$$

Let X be an arbitrary element of $\wp(A) \cup \wp(B)$. We must prove that X belongs to $\wp(A \cup B)$.

If X belongs to of $\wp(A) \cup \wp(B)$, either X belongs to of $\wp(A)$ or X belongs to of $\wp(B)$.

Case 1: X belongs to $\wp(A)$. By definition of power set, $X \subseteq A$. By definition of union, A $\subseteq A \cup B$. (Anything in A must be in A union B!) The subset relationship is transitive. If A is a subset of B and B is a subset of C, then A is a subset of C. This means that X must be a subset of $A \cup B$. By definition of power set, $X \in \wp(A \cup B)$.

Case 1: X belongs to $\wp(B)$. By definition of power set, $X \subseteq B$. By definition of union, A $\subseteq A \cup B$. (Anything in B must be in A union B!) This means that X must be a subset of $A \cup B$. By definition of power set, $X \in \wp(A \cup B)$.

Questions:

What happens with intersection, what happens if we try to flip the relationship in this question?