COP 3503 10/6/26

1) Exam Thursday, can come early
Jackson will give the exam
2) I'm out of town W-F.
3) Exam Review

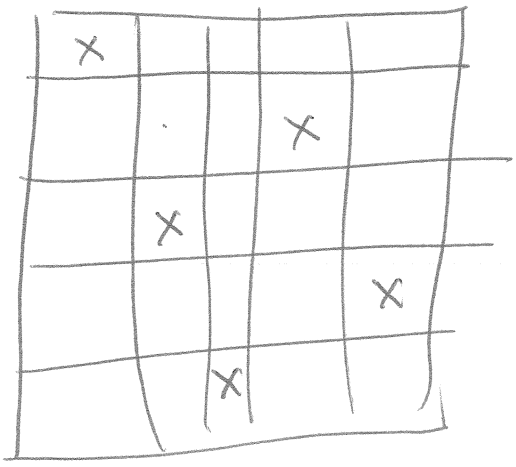
Rolling Median Problem

6, 2, 9, 15, 1, 7, 8, 12, 13, 16

[6]	[ ]	6×2=12
[2 6]	[ ]	
[2]	[6]	2+6=8
2	[6 9]	6×2=12
[2]	[6, 9, 15]	6+9=15
[2, 6]	[9, 15]	

etc.

5 & Queens

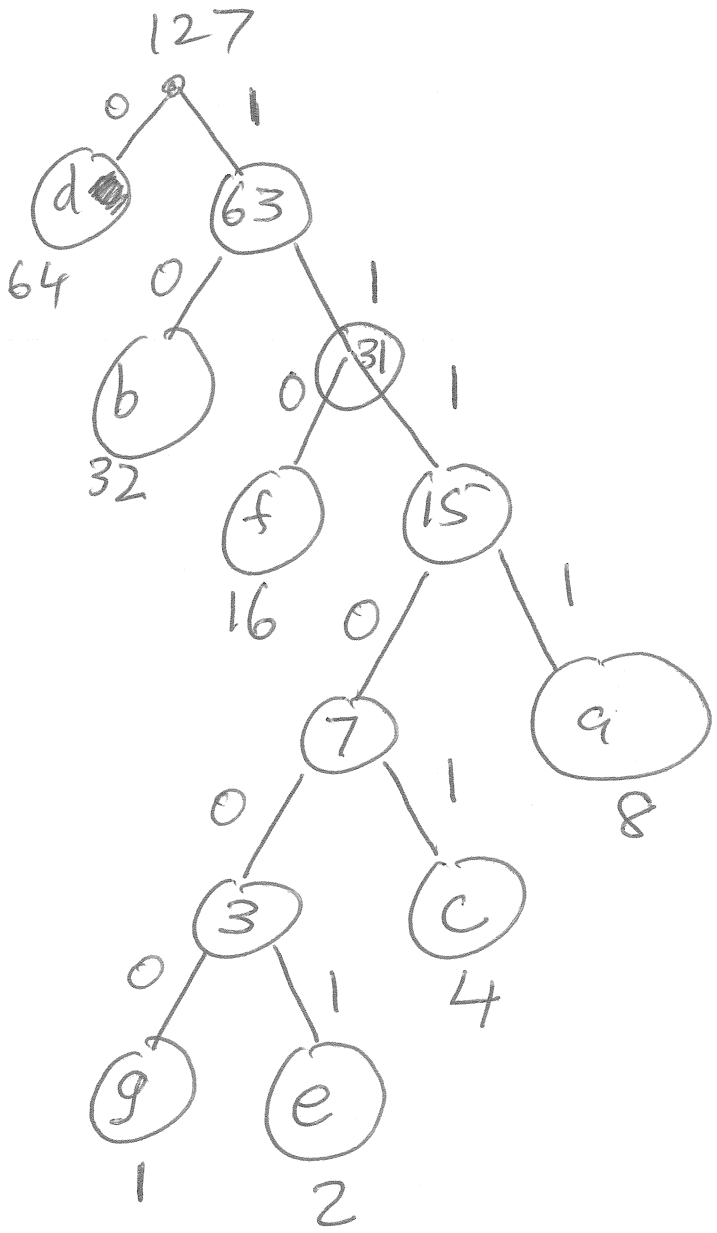


Prefix Composite

```
if (k==n) {
    S.o.p(cur);
    return;
}
int start = k==0? 1 : 0;
for (int i=start; i<10; i++) {
    int newnum = 10*cur + start;
    if (isprime(newnum)) continue;
    go(newnum, k+1, n);
}
```

Huffmen

✓ a	1111	8
✓ b	10	32
✓ c	11101	4
✓ d	0	64
✓ e	111001	2
✓ f	110	16
✓ g	111000	1



Coffee

Sort most to least profit for each type take as many cups as you can. You might get to a type where you run out of cups so take what you can and break out.

Coffee

Profit 10	P 8	P 7	P 5	P 4
# 6	#10	#3	#6	#4

6×10 8×10 3×7 3×5 22

60 80 21 15 cups

break

$22 - 6 = 16 - 10 = 6 - 3 = 3$

Avg # of nodes w/ 2 children in BST

root	prob	# nodes w/ 2 children
		$T(n-1)$
1	$\frac{1}{n}$	$1 + T(1) + T(n-2)$
2	$\frac{1}{n}$	$1 + T(2) + T(n-3)$
3	$\frac{1}{n}$	
⋮		⋮
		$1 + T(n-2) + T(1)$
n	$\frac{1}{n}$	$T(n-1)$



$$T(n) = \frac{1}{n}\left[\left(2\sum_{i=1}^{n-1} T(i)\right) + (n-2)\right]$$

$$T(n) = \left(\frac{2}{n}\sum_{i=1}^{n-1} T(i)\right) + \left(\frac{n-2}{n}\right)$$

$$T(n) = \frac{2}{n}\sum_{i=1}^{n-1} T(i) + \left(\frac{n-2}{n}\right)$$

$$n\,T(n) = \left(2\sum_{i=1}^{n-1} T(i)\right) + (n-2)$$

$$-(n-1)\,T(n-1) = \left(2\sum_{i=1}^{n-2} T(i)\right) + (n-3)$$

$$n\,T(n) - (n-1)\,T(n-1) = 2\,T(n-1) + 1$$

$$\frac{n\,T(n)}{n(n+1)} = \frac{(n+1)\,T(n-1)}{n(n+1)} + \frac{1}{n(n+1)}$$

$$\frac{T(n)}{n+1} = \frac{T(n-1)}{n} + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

Note:
$T(0) = 0$
$T(1) = 0$
$T(2) = 0$
$T(3) = \frac{1}{3}$

Let $S(n) = \frac{T(n)}{n+1}$

$S(3) = \frac{T(3)}{4}$

$S(3) = \frac{1/3}{4} = \frac{1}{12}$

$$S(n) = S(n-1) + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S(n) = S(k) + \sum_{i=k+1}^{n}\left(\frac{1}{i} - \frac{1}{i+1}\right)$$

Note: My error was using a base case that was invalid. $S(2)$ is smallest that works.

$$S(n) = S(3) + \frac{1}{4} - \frac{1}{n+1}$$

$$S(n) = \frac{1}{12} + \frac{1}{4} - \frac{1}{n+1} = \frac{1}{3} - \frac{1}{n+1}$$

$$S(n) = \frac{T(n)}{n+1}$$

$$\frac{1}{3} - \frac{1}{n+1} = \frac{T(n)}{n+1}$$

$$T(n) = (n+1)\left(\frac{1}{3} - \frac{1}{n+1}\right) = \frac{n+1}{3} - \frac{3}{3} = \boxed{\frac{n-2}{3}}$$