

- 0 0 1 1 0 0 0
- 1) Insert 20, 10, 5, 7, 2, 15, 30
1 2 3-A
- 2) Delete 2, 5, 7
- 0 0 1 2
- 3) Insert 5, 35, 40, 33
- 1 3-B
- 4) Delete 33, 30
2
- 5) Insert 13
- 2 3-C
- 6) Delete 35, 40

R-B tree
picture
example

9/17/26

Shown
beginning
of class

Cases 1: 0, 0, 1,

COP 3503 - 9/17/26

✓ (1) R-B note edit

✓ (2) Example

(3) Binary Index Tree (Fenwick Tree)

Two Operations on an array

1) Add x to index i] $O(1)$

2) Query: Sum of values from
index i to index j . $O(n)$

1	2	3	4	5	6	7					n
			7			8					

array
size = n

Add 7 to index 4

Add 8 to index 7 etc.

Sum (2, 1000) $\rightarrow$ 15

Prefix Sum (Cumulative frequency array)

if my c.f. array index i stores
sum of all values in original array
from index 1 to index i .

0 0 0 7 7 7 15 15 ...

$$\text{Sum}(i, j) = \text{ps}[j] - \text{ps}[i-1]$$

$$\begin{aligned} & \left(\begin{array}{c} \swarrow \\ X_1 + X_2 + X_3 + \dots + X_i + X_j \\ \searrow \end{array} \right) \\ & - X_1 - X_2 - X_3 - \dots - X_{i-1} \end{aligned}$$

Operation #1: Add x to index i $O(n)$

#2: Query $O(1)$

#2 = Query $O(1)$

	5	8	8	8	8	12	12	12	14	14	
pf	0	0	3	3	3	3	7	7	7	9	9
orig	0	0	5	3	0	0	0	4	0	0	2
index	1	2	3								

add
5 to
index 2

orig 0 0 3 0 0 4 0 0 2 0

index	1	2	3
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Chart of Responsibility

Size of responsibility	BIT index	Orig Array Index List
1	1	1
2	2	1, (2)
3	3	3
4	4	1, (2), 3, 4
5	5	[5]
6	6	[5], 6
7	7	7
8	8	1, (2), 3, 4, [5], 6, 7, 8

$tot += bit[i]$
 $i = \text{lowest_one_bit}(i)$

Add

x to i $\nearrow$
 $bit[i] += x$
 $i += lowestonebit(i)$

Query: Sum from 1 to i

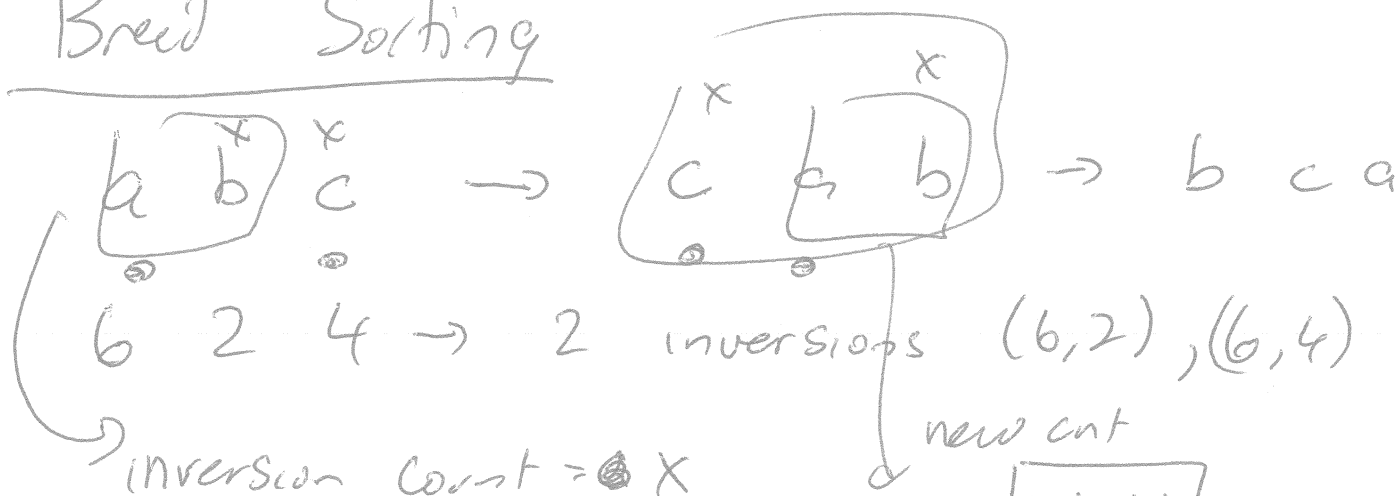
Consider $\rightarrow$

$$\text{lowestonebit}(i) = i \& (-i)$$

4 bits

0011	)	= 3
1101	)	= -3
↓ ↓ ↓		
-8 + 4 + 1		

Bread Sorting



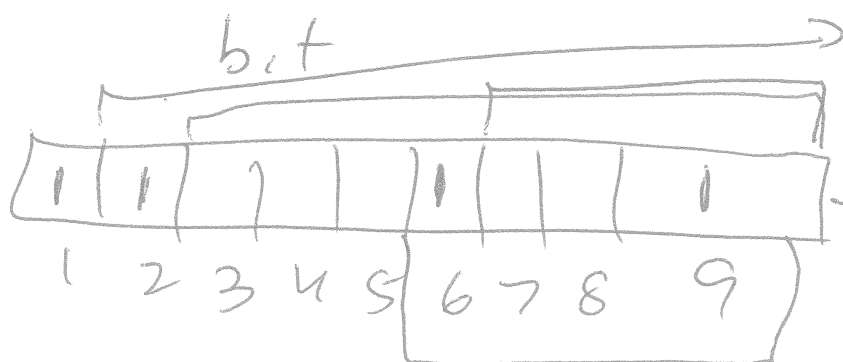
new cnt
 $= X \begin{bmatrix} \pm & | & \pm & | \end{bmatrix}$

ac
 $\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$

bread sort operation
 leaves the # parity
 of the # of inversions
 UNCHANGED!

Inversion Counting w/ BIT in $O(n \log n)$

6, 2, 9, 1, 5
 ↑ ↑ ↑



Counter

$6 \rightarrow 0 \checkmark$
 $2 \rightarrow 1 \checkmark$
 $9 \rightarrow 0 \checkmark$
 $1 \rightarrow 3 \checkmark$
 $5 \rightarrow 2 \checkmark$

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