Summation Formula Sheet

(1) Can take out constants:

$$\sum_{k=a}^{b} c \longrightarrow c\sum_{k=a}^{b} 1$$

$$\sum_{i=1}^{n} 5n^2 i \longrightarrow 5n^2 \sum_{i=1}^{n} i$$

(2) How to sum a constant:

$$\sum_{k=a}^{b} 1 = (b-a+1)$$

(3) $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

← Formula for $1+2+3+\cdots+n$

(4) $$\sum_{k=1}^{n} (2k+1) = \sum_{k=1}^{n} 2k + \sum_{k=1}^{n} 1$$

Splitting up terms

(5) Make the index start from 1:

$$\sum_{k=5}^{n} k = \sum_{k=1}^{n} - \sum_{k=1}^{4} k$$

Find the closed form solution of:

$$\sum_{i=15}^{100} 8i + 12$$

$$= \sum_{i=15}^{100} \underline{\underline{8}}\, i + \sum_{i=15}^{100} 12$$

$$= \underbrace{8 \sum_{i=15}^{100} i} + 12(100 - 15 + 1)$$

$$+ 12 \cdot 86$$

$$= 8\left(\sum_{i=1}^{100} i - \sum_{i=1}^{14} i \right) + 12 \cdot 86$$

$$\boxed{\frac{n(n+1)}{2} = \sum_{i=1}^{n} i}$$

$$= \overset{4}{\cancel{8}}\left(\frac{100(101)}{\cancel{2}} - \frac{14(15)}{\cancel{2}} \right) + 12 \cdot 86$$

$$= 4(10100 - 210) + 12 \cdot 86 =$$

Closed form solution of:

$$\sum_{i=n+1}^{2n} (2i + 3n^2)$$

$$\sum_{i=n+1}^{2n} 2i + \boxed{\sum_{i=n+1}^{2n} \underbrace{3n^2}_{\text{constant}}}$$

$$+ (\cancel{2}n - \cancel{(n+1)} \cancel{+1}) 3n^2$$

$$2\sum_{i=n+1}^{2n} i + 3n^3$$

$$2\left(\sum_{i=1}^{2n} i - \sum_{i=1}^{n} i\right) + 3n^3$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

$$\cancel{2}\left(\frac{2n(2n+1)}{\cancel{2}} - \frac{n(n+1)}{\cancel{2}}\right) + 3n^3$$

$$3\cancel{4}n^2 + \cancel{2}n - \cancel{n^2} - \cancel{n} + 3n^3$$

$$= \boxed{3n^3 + 3n^2 + n}$$

$$\sum_{i=0}^{n}\left[2\sum_{j=n+1}^{3n}(i+j)\right]$$ Find the closed form solution of

$$\Downarrow$$

$$2\left(\sum_{j=n+1}^{3n} i + \sum_{j=n+1}^{3n} j\right) \qquad \sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

$$\Downarrow$$

$$2i(\cancel{3n}^{2n} - \cancel{(n}+1)+1) + \sum_{j=1}^{3n} j - \sum_{j=1}^{n} j$$

$$2 \cdot i \cdot 2n \qquad \frac{9n^2+3n}{2} \quad - n^2 \cancel{+} n$$

$$4ni + \frac{3n(3n+1)}{\cancel{2}} - \frac{n(n+1)}{\cancel{2}}$$

$$\sum_{i=0}^{n}\left[\underbrace{4ni}_{} + \underbrace{(8n^2-2n)}_{}\right]$$

$$\boxed{4n\left(\frac{n(n+1)}{2}\right) + (8n^2-2n)(n+1)}$$

$$10n^3 + 12n^2 + 2n$$