

2) (10 pts) Write a function that takes in a pointer to a linked list and uses it to create a new linked list that stores the first, third, fifth, etc. values in the input linked list in the original order, returning a pointer to the front of this newly created list. **Write your function recursively.** (For example, if the input list to the function stores 2, 6, 5, 7, 3, 9 then your function will create a new list storing 2, 5, 3, returning a pointer to the node storing 2. If the input list stored 12, 10, 13, then your function will create a new list storing 12, 13, returning a pointer to the node storing 12.) Watch out for NULL pointers!!!

```
typedef struct node {
    int data;
    struct node* next;
} node;
```

```
node* oddrankedterms(node* front) {
```

```
    if (front == NULL) // front->next == NULL
        return front;
```

```
    if (front->next == NULL) {
```

```
        node* res = malloc(sizeof(node));
```

```
        res->data = front->data;
```

```
        res->next = NULL;
```

```
        return res;
```

```
    } // No more to add
    if (front->next == NULL) ] Special case
        res->next = NULL;    avoids NULL ptr
                             error.
```

```
    else // this is safe.
```

```
        res->next = oddrankedterms(res->next->next);
```

```
    return res;
```

```
}
```

6) (10 pts) Please give an exact solution (a function that  $T(n)$  satisfies in terms of  $n$ ) to the following recurrence relation using the iteration technique:

$$T(n) = T(n-1) + n2^n, \text{ for } n > 1$$

$$T(1) = 0$$

$$T(n) = T(n-1) + n \cdot 2^n$$

$$= T(n-2) + (n-1)2^{n-1} + n2^n$$

$$= \underline{T(n-3)} + \underline{(n-2)2^{n-2}} + \underline{(n-1)2^{n-1}} + \underline{n2^n}$$

$$T(n-1) = T(n-2) + \cancel{(n-1)2^{n-1}} + (n-1)2^{n-1}$$

$$T(n-2) = T(n-3) + (n-2) \cdot 2^{n-2}$$

After  $k$  iterations

$$T(n) = T(n-k) + \sum_{i=n-k+1}^n i \cdot 2^i$$

Let  $n-k=1$ , so  $k=n-1$ , plug into the recurrence:

$$T(n) = T(1) + \sum_{i=2}^n i \cdot 2^i = \sum_{i=2}^n i \cdot 2^i \quad \text{Let } S = \sum_{i=2}^n i \cdot 2^i$$

$$S = \boxed{2 \cdot 2^2} + \boxed{3 \cdot 2^3} + 4 \cdot 2^4 + \dots + \boxed{n \cdot 2^n}$$

$$-2S = 2 \cdot 2^3 + 3 \cdot 2^4 + \dots + (n-1) \cdot 2^n + n \cdot 2^{n+1}$$

$$-S = \boxed{2 \cdot 2^2} + 1 \cdot 2^3 + 1 \cdot 2^4 + \dots + 1 \cdot 2^n - n \cdot 2^{n+1}$$

$$-S = (1 + 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n) - n \cdot 2^{n+1}$$

$$-S = 1 + \sum_{i=0}^n 2^i - n \cdot 2^{n+1}$$

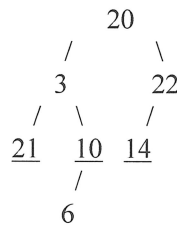
$$-S = 1 + 2^{n+1} - n \cdot 2^{n+1}$$

$$S = n \cdot 2^{n+1} - 2^{n+1} = \boxed{(n-1)2^{n+1}}$$

7) (6 pts) Show the contents of the following array after each merge in Merge Sort.

Original	17	13	7	19	22	2	15	14
After 1 <sup>st</sup>								
After 2 <sup>nd</sup>								
After 3 <sup>rd</sup>								
After 4 <sup>th</sup>								
After 5 <sup>th</sup>								
After 6 <sup>th</sup>								
Last	2	7	13	14	15	17	19	22

8) (10 pts) Consider the following problem: given a pointer to the root of a binary tree, and a non-negative integer, k, find the sum of all the values in the nodes that are exactly k levels below the root node. For example, in the tree below, the sum of the values 2 levels below the root is  $21 + 10 + 14 = 45$ .



Write a function that takes in a pointer to the root of a binary tree and the value of k and returns this sum. Your function **must be recursive**.

```

typedef struct treenode {
    int data;
    struct treenode* left;
    struct treenode* right;
} treenode;

```

```

int sumdepthk(treenode* root, int k) {

```

```

    if (root == NULL) return 0;
    if (k == 0) return root->data;
    int res = 0;
    res += sumdepthk(root->left, k-1);
    res += sumdepthk(root->right, k-1);
    return res;

```

```

}

```

10) (10 pts) Consider the task of printing out the (in lowercase letters) the first word alphabetically stored in a trie. (You may assume if a link exists in a trie, that some word exists further down that link.) Write a void recursive function to accomplish this task. (Note: Different recursive calls will print different letters in this word.)

```
typedef struct trienode {
    int isWord;
    struct trienode* next[26];
} trienode;

void printfirst(trienode* root) {
```

if (root → isWord) return;

for (int i = 0; i < 26; i++) {

if (root → next[i] == NULL)  
continue;

// What is true if we get here?

// A word ~~does~~ uses this letter link.

printf("%c", 'a' + i);

printfirst(root → next[i]);

break; // important to force only 1 word print.

}



11) (5 pts) Convert 782 in base 10 to base 4.

12) (10 pts) There are  $n$  teams in a software competition. Each team has  $m$  students. Each student knows some subset of programming languages. There are 20 recognized languages for the competition, numbered 0 to 19, so the subset of languages a single student knows can be stored in a single integer bitmask, which indicates that a student knows language  $i$ , if and only if bit  $i$  is set to 1. A team can complete a task that requires language  $i$  as long as at least one of its team members knows language  $i$ . Write a function that takes in an array, `languages`, where `languages[i][j]` stores an integer bitmask representing the set of languages that student  $j$  on team  $i$  knows, as well as the previously mentioned values of  $n$  and  $m$ , and returns a single integer bitmask storing the set of languages such that all teams could complete a task with that particular language. (Thus, if all teams have at least one individual who knows languages 0, 1, 3 and 5, and the same statement can't be made about any other language, then the function should return  $101011_2 = 44$ .)

```
int getwinmove(int** languages, int n, int m) {
```

```
    // Initialize to all languages.
    int res = (1<<20)-1;
```

```
    /* Fill in code here. */
```

```
    for (int i = 0; i < n; i++) {
```

```
        int team = 0;
```

```
        for (int j = 0; j < m; j++)
```

```
            team |= languages[i][j];
```

```
        res &= team;
```

```
    }
```

```
    return res;
```

```
}
```

	0	1	2	3	all
0	010	100	001	111	(111)
1	✓	✓	✓	✓	(101)
2	✓	✓	✓	✓	(001)

↓  
1010

OR will  
get all  
languages covered  
by team

res = 111 (7)

team = 1018 (5) x  
101 35

785=5

100011

13) (3 pts) After which American linguist/computer scientist/philosopher is Chomsky Normal Form named after? (Note: He was born on December 7, 1928 and is still alive!)