

WOP 3502 10/15/25

$$T(n) = \boxed{3T(n-1) + 1}, \quad T(1) = 1$$

$$= 3[3T(n-2) + 1] + 1$$

$$= \boxed{9T(n-2) + (3+1)}$$

$$= 9[3T(n-3) + 1] + (3+1)$$

$$= \boxed{27T(n-3) + (1+3+9)}$$

$$T(n-1) = 3T(n-2) + 1$$

$$T(n-2) = 3T(n-3) + 1$$

After k iterations

$$T(n) = 3^k T(n-k) + \sum_{i=0}^{k-1} 3^i$$

We know $T(1)$, let $n-k=1$, $k=n-1$

Plug in $k=n-1$:

$$T(n) = 3^{n-1} T(1) + \sum_{i=0}^{n-2} 3^i$$

$$= 3^{n-1} + \sum_{i=0}^{n-2} 3^i$$

$$= \sum_{i=0}^{n-1} 3^i = \frac{3^n - 1}{3 - 1} = \boxed{\frac{3^n - 1}{2}}$$

$$a + ax + ax^2 + ax^3 + \dots + ax^{n-1} = a \left(\frac{x^n - 1}{x - 1} \right), \quad x \neq 1$$

$$\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}, \quad x \neq 1.$$

Sorting 10/15/25

Strongly recommend coding all 3 of these on your own!

$O(n^2)$ sorting

- WORST CASE

- 1) Bubble Sort
- 2) Insertion Sort
- 3) Selection Sort

Bubble Sort

6 3 2 9 5 4

3 6 2 9 5 4

3 2 6 9 5 4

3 2 6 9 5 4

3 2 6 5 9 4

3 2 6 5 4 9 end iter 1

2 3 6 5 4 9

2 3 6 5 4 9

2 3 5 6 4 9

2 3 5 4 6 9 end iter 2

2 3 5 4 6 9

2 3 5 4 6 9

2 3 4 5 6 9 end iter 3

2 3 4 5 6 9

2 3 4 5 6 9

1 2 3 4 5 6 9

```
for (int i = n-1; i > 0; i--)
```

```
for (int j = 0; j < i; j++)
```

```
if (a[j] > a[j+1])
```

```
swap(a, j, j+1)
```

end iter 1

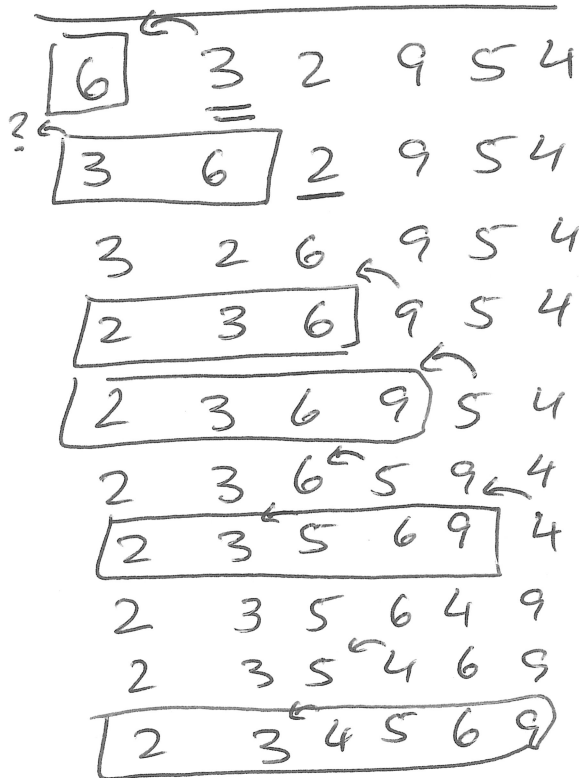
end iter 2

end iter 3

end iter 4

end iter 5

Insertion Sort



Sorted array size 1, insert 3

End of iter 1

End of iter 2

End of iter 3

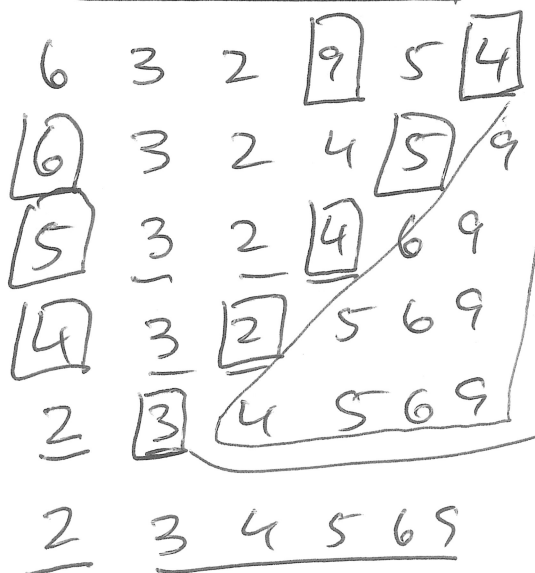
End of iter 4

End of iter 5

```

for (int i = 1; i < n; i++) {
    int j = i;
    while (j > 0 && a[j] < a[j-1]) {
        swap(a[j], a[j-1]);
        j--;
    }
}
    
```

Selection Sort



only 1 swap per iteration

maxi = 3

end iter 1, maxi = 0

end iter 2, maxi = 0

end iter 3, maxi = 0

end iter 4, maxi = 1

Swap with itself end iter 5

```

for (int i = n-1; i > 0; i--) {
    int maxi = 0;
    for (int j = 1; j <= i; j++)
        if (a[j] > a[maxi])
            maxi = j;
    swap(a, maxi, i);
}
    
```

My versions

	<u>Best</u>	<u>Avg</u>	<u>Worst</u>	
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$	$\sum_{i=1}^{n-1} i = \frac{(n-1)n}{2} = O(n^2)$
Insertion	$O(n)$	$O(n^2)$	$O(n^2)$	Avg case is $\frac{(n-1)n}{4} = O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$	