COP 3502 10/15/25

$$T(n) = \boxed{3T(n-1)+1}, \quad T(1)=1$$

$$= 3[3T(n-2)+1]+1$$

$$= \boxed{9T(n-2) + \boxed{(3+1)}}$$

$$= 9[3T(n-3)+1] + (3+1)$$

$$= \boxed{27T(n-3) + (1+3+9)}$$

$T(n-1) = 3T(n-2)+1$

$T(n-2) = 3T(n-3)+1$

After k iterations

$$T(n) = 3^k T(n-k) + \sum_{i=0}^{k-1} 3^i$$

We know $T(1)$, let $n-k=1$, $k=n-1$

Plug in $k=n-1$:

$$T(n) = 3^{n-1} T(1) + \sum_{i=0}^{n-2} 3^i$$

$$= 3^{n-1} + \sum_{i=0}^{n-2} 3^i$$

$$= \sum_{i=0}^{n-1} 3^i = \frac{3^n - 1}{3-1} = \boxed{\frac{3^n-1}{2}}$$

$$a + ax + ax^2 + ax^3 + \cdots ax^{n-1} \qquad \left(\frac{x^n-1}{x-1}\right), \quad x \neq 1$$

$$\underline{a(1 + x + x^2 + x^3 + \cdots x^{n-1})} = a\left(\frac{x^n-1}{x-1}\right)$$

$$\sum_{i=0}^{n-1} x^i = \frac{x^n-1}{x-1}, \quad x \neq 1.$$

Sorting 10/15/25

Strongly recommend coding all 3 of these on your own!

$O(n^2)$ sort~~ing~~s — WORST CASE

1) Bubble Sort
2) Insertion Sort
3) Selection Sort

Bubble Sort

```
for(int i=n-1; i>0; i--)
  for(int j=0; j<i; j++)
    if (a[j] > a[j+1])
      swap(a, i, j)
```

6 3 2 9 5 4

3 6 2 9 5 4

3 2 6 9 5 4

3 2 6 9 5 4

3 2 6 5 9 4

3 2 6 5 4 $\boxed{9}$ end iter 1

2 3 6 5 4 9

2 3 6 5 4 9

2 3 5 6 4 9

2 3 5 4 $\boxed{6 \quad 9}$ end iter 2

2 3 5 4 6 9

2 3 5 4 6 9

2 3 4 $\boxed{5 \quad 6 \quad 9}$ end iter 3

2 3 4 5 6 9

2 3 $\boxed{4 \quad 5 \quad 6 \quad 9}$ end iter 4

$\boxed{2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 9}$ end iter 5

Insertion Sort

6 3 2 9 5 4 — sorted array size 1, insert 3

3 6 2 9 5 4 — End of iter 1

3 2 6 9 5 4

2 3 6 9 5 4 — End of iter 2

2 3 6 9 5 4 — End of iter 3

2 3 6 5 9 4

2 3 5 6 9 4 — End of iter 4

2 3 5 6 4 9

2 3 5 4 6 9

2 3 4 5 6 9 — End of iter 5

for(int i=1; i<n; i++){
  int j=i;
  ~~while (j>0){~~
  ~~if (a[j]~~
  while(j>0 && a[j] < a[j-1])
  { ~~j~~
    swap(a, j, j-1);
    j--;
  }
}

Selection Sort

only 1 swap per iteration

6 3 2 9 5 4 — maxi = ~~0~~ 3

6 3 2 4 5 9 — end iter 1, maxi=0

5 3 2 4 6 9 — end iter 2 maxi=0

4 3 2 5 6 9 — end iter 3 maxi=0

2 3 4 5 6 9 — end iter 4 maxi= ~~0~~ 1

2 3 4 5 6 9 — swap with itself end iter 5

for (int i=n-1; i>0; i--) {
  ~~max~~ int maxi = 0;
  for (int j=1; j<=i; j++)
    if (a[j] > a[maxi])
      ~~swap (a,~~ maxi = j
  swap (a, maxi, i);
}

My versions

	Best	Avg	Worst
Bubble	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion	$O(n)$	$O(n^2)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$

$$\sum_{i=1}^{n-1} i = \frac{(n-1)n}{2} = O(n^2)$$

Avg case is

$$\frac{(n-1)n}{4} = O(n^2)$$