COP 3502 10/13/25

Sums continued → purpose to help determine Big-Oh run time of iterative code.

$$\sum_{i=a}^{b} c = c(b-a+1)$$

$a \le b$

$$c + c + c + \cdots + c$$
$$\uparrow \quad \uparrow \quad \uparrow \quad \quad \uparrow$$
$$a \quad a+1 \quad a+2 \quad \quad b$$

$$S = \sum_{i=1}^{n} i = 1 + \boxed{2} + 3 + \cdots + n$$
$$\quad\quad n + \boxed{(n-1)} + (n-2) + \cdots + 1$$

$$S = \sum_{i=1}^{n} (n+1-i) = 100 + 99$$

$$\begin{array}{r} 1 \\ +2 \\ \hline 3 \\ +3 \\ \hline 6 \\ +4 \\ \hline 10 \\ \vdots \\ \boxed{} \\ +98 \end{array}$$

$$2S = \sum_{i=1}^{n} i + (n+1-i) \quad \underbrace{(n+1) \quad (n+1) \quad (n+1) \quad \cdots + (n+1)}$$

how many times is $n+1$ added?

Ans $= n$

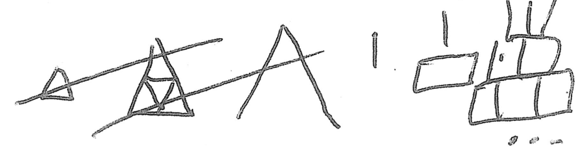
$$2S = \sum_{i=1}^{n} (n+1)$$

$$\frac{n(n+1)}{2} \leftarrow \text{each \# added twice}$$

$$2S = n(n+1)$$

$$S = \frac{n(n+1)}{2}$$

$1, 3, 6, 10, \ldots$ (triangle numbers...)



$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=a}^{b} f(i) = \sum_{i=1}^{b} f(i) - \sum_{i=1}^{a-1} f(i)$$

$1 < a \le b$

$$f(a) + f(a+1) + f(a+2) + \cdots f(b) \quad \Big\| \quad \cancel{f(1)} + \cancel{f(2)} + \cdots \cancel{f(a-1)} + \boxed{f(a) + f(a+1) \cdots f(b)}$$

$n=3$

$$\cancel{\sum_{i=4}^{6} i = 4+5+6 = 15}$$

$$\cancel{\frac{3 \cdot 6}{2}(2 \cdot 3^2 - 1) = \frac{3}{2}(17)}$$

$$\sum_{i=n+1}^{2n} i = \sum_{i=1}^{2n} i - \sum_{i=1}^{n} i$$

$$= \frac{(2n)(2n+1)}{2} - \frac{n(n+1)}{2}$$

$$= \frac{n}{2}\left(\cancel{2}^{\,2}(2n+1) - (n+1)\right)$$

$2n+1-n-1$

$$\cancel{= \frac{n}{2}\left(2n^2 + n - n - 1\right)}$$

$$\cancel{= \boxed{\frac{n}{2}\left(\frac{2n^2 - 1}{n}\right)} = \frac{n^2}{2}}$$

$$= \frac{n}{2}(4n + 2 - n - 1)$$

$$= \frac{n}{2}(3n+1) \longrightarrow \frac{3}{2}(3 \cdot 3 + 1) = 15 \checkmark$$

$$\sum_{i=a}^{b}(f(i) + g(i)) = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$

$f(a) + g(a)$
$f(a+1) + g(a+1)$
$\vdots$
$f(b) + g(b)$

$f(a) + f(a+1) + f(a+2) + \cdots f(b)$

$g(a) + g(a+1) + g(a+2) \cdots g(b)$

$$\sum_{i=a}^{b} f(i) \times g(i) \neq \left(\sum_{i=a}^{b} f(i)\right) \times \left(\sum_{i=a}^{b} g(i)\right)$$

$$\boxed{\begin{array}{l} f(a)g(a) + \\ f(a+1)g(a+1) + \end{array}}$$

$f(a+2)g(a+2) +$

$+$

$\vdots$

$f(b)g(b)$

$(f(a) + f(a+1) + f(a+2) + \cdots + f(b)) \times$
$(g(a) + g(a+1) + \cdots \; g(b))$

$$(f(a) + f(a+1))(g(a) + g(a+1))$$
$$= f(a)g(a) + f(a)g(a+1) + f(a+1)g(a) + f(a+1)g(a+1)$$

factor rule

$$\boxed{\sum_{i=a}^{b} c f(i) = c \sum_{i=a}^{b} f(i)}$$

FOIL

$$\sum_{i=n-1}^{2n+3} (3i-5) = \sum_{i=1}^{2n+3} (3i-5) - \sum_{i=1}^{n-2} (3i-5)$$

$$= \sum_{i=1}^{2n+3} 3i - \sum_{i=1}^{2n+3} 5 - \left(\sum_{i=1}^{n-2} 3i - \sum_{i=1}^{n-2} 5\right)$$

$$= 3\sum_{i=1}^{2n+3} i - 5(2n+3) - 3\sum_{i=1}^{n-2} i + 5(n-2)$$

$$= \frac{3(2n+3)(2n+4)}{2} - 10n - 15 - \frac{3(n-2)(n-1)}{2} + 5n - 10$$

$$= \frac{3(2n+3)\cancel{2}(n+2)}{\cancel{2}} - \frac{3(n-2)(n-1)}{2} - 5n - 25$$

$$= 3(2n^2 + 7n + 6) - \frac{3}{2}(n^2 - 3n + 2) - 5n - 25$$

$$= 6n^2 + 21n + 18 - \frac{3}{2}n^2 + \frac{9}{2}n - 3 - 5n - 25$$

$$= \frac{9}{2}n^2 + \frac{41}{2}n - 10$$

$$S = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \cdots$$ Infinite geo sum

$$S = a + \cancel{ax} + \cancel{ax^2} + \cancel{ax^3} + \cdots \qquad |x| < 1$$

$$-xS = \cancel{ax} + \cancel{ax^2} + \cancel{ax^3} + \cdots$$

$$S - xS = a$$

$$S(1-x) = a$$

$$S = \frac{a}{1-x}, \quad \text{infinite geo sum} \quad \frac{1^{st}\text{ term}}{1 - \text{common ratio}}$$

$$S = \sum_{i=0}^{n-1} a x^i = a + \cancel{ax} + \cancel{ax} + \cdots \cancel{ax^{n-1}}$$

$$-xS = \quad = \cancel{ax} + \cancel{ax^2} + \cdots \cancel{ax^{n-1}} + ax^n$$

$$S - xS = a - ax^n$$

$$S(1-x) = a(1-x^n)$$

$$S = \frac{a(1-x^n)}{1-x} = \frac{a(x^n-1)}{x-1}, \quad x \neq 1.$$

```
for (int i = 0; i<n; i++)
  for (int j=0; j<m; j++)
    sum++;
```

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{m} 1\right)$$

$$\sum_{i=1}^{n} m = nm$$

$$= \sum_{i=1}^{n} m$$

$O(nm)$

$$= nm$$

```
for (int i=0; i<n; i++) {
  for (int j=0; j<i; j++) {
    if ( ) {

    }
  }
}
```

Const

$$\sum_{i=0}^{n-1}\left(\sum_{j=0}^{i-1} 1\right)$$

$$= \sum_{i=0}^{n-1} i = \boxed{\frac{(n-1)n}{2}}$$

$$= O(n^2)$$

```
i=n, res=0
while (n>0) {
  res = res + n%2;
  n = n/2;
}
```