COP 3502 10/1/25

0) Hand Back Papers
1) Programs - AIDS (Bunch P1, P2 → O over next few days)

Please do your own work!

2) $ ULA Meeting - upto 16 private tutoring sessions a week (currently ~ 6)
- Ryan + Colin (appts for 2 weeks)

3) Timing Problems (Lab Sheet)

Why is it useful to "know" the Big-Oh runtime of algorithm?

Predict how long an algorithm will take for different input sizes.

Why we use Big-Oh instead of exact functions?
→ When an <u>algorithm</u> gets implemented

Different Architectures
Different Clock Speeds
...

Problem

Alg sorts #s in $O(n^2)$ time.
For $n = 10,000$ it takes 1.3 sec.
How long do we expect it to take to sort
$n = 40,000$ items?

Let $T(n)$ = run-time input size n
$= cn^2$, for some constant c.

Given $T(10000) = c\,(10000)^2 = 1.3\,s$

$$c = \frac{1.3\,s}{(10^4)^2}$$

$$\boxed{\begin{array}{l} \frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2 \\ \frac{a^k}{b^k} = \left(\frac{a}{b}\right)^k \end{array}}$$

Find $T(40000) = c \cdot 40000^2$

$$= \frac{1.3\,s}{(10000)^2} \times (40000)^2 = (1.3s) \times \left(\frac{40000}{10000}\right)^2$$

$$= 1.3s \times 16$$

$$= \cancel{2.08}\ \boxed{20.8 \text{ sec}}$$

$$\begin{array}{r} 16 \\ 13 \\ \hline 48 \\ 16 \end{array}$$

Improved mult algorithm to multiply 2 polynomials of degree n in $O(n\sqrt{n})$ time.
For $n = 10^4$, it runs in 30 ms. How long will it take for $n = 160,000$?, in seconds?

Let $T(n) = cn\sqrt{n}$, c is const be run-time of alg.

Given $T(10^4) = c\,10^4\sqrt{10^4} = 30\text{ ms}$

$$c(10^4)(10^2) = 30\text{ ms}$$

$$c = \frac{30\text{ ms}}{10^6}$$

Find $T(160,000) = T(16 \times 10000)$

$$= c \cdot 16 \times 10000 \sqrt{16 \times 10000}$$

$$= \frac{30\text{ ms}}{\cancel{10^6}} \times 16 \times \cancel{10^4} \times 4 \times \cancel{10^2}$$

$$= 120\text{ ms} \times 16$$

$$= 1920\text{ ms}$$

$$= \boxed{1.92\text{ seconds}}$$

An algorithm with a run time of $O(n!)$ takes 100 ms to run on an input of size 9.
On a different input size it took it took 11 seconds
How big was that input?

Let $T(n) = cn!$ be the alg run time

$$T(9) = c\,9! = 100 \text{ ms}$$

$$c = \frac{100 \text{ ms}}{9!}$$

$$T(x) = c\,x! = 11000 \text{ ms}$$

$$\frac{100 \text{ ms}}{9!} \cdot x! = 11000 \text{ ms}$$

$$\frac{x!}{9!} = 110 \text{ ms} \qquad \frac{1 \cdot 2 \cdot 3 \cdots x!}{1 \cdot 2 \cdot 3 \cdots 9!}$$

formal

$$\prod_{i=10}^{x} i \qquad \overbrace{10 \cdot 11 \cdots x} = 110 \text{ ms}$$

$$11 \cdots x = 11 \text{ ms}$$

works out exactly for $x = 11$.

CountInv

$[1.5, 2.5)$

low 13.5sec

high 22.5sec

$$T(30000) = c \cdot 30000^2 = 2 \text{sec}$$

$$c = \frac{2}{30000^2}$$

$$T(3 \cdot 30000) = \frac{2}{30000^2} \times (3 \times 30000)^2 = \frac{2 \times 3^2 \times \cancel{30000^2}}{\cancel{30000^2}}$$

$$= 18 \text{ sec}$$

Count Inv

90000 $\rightarrow$ 23 sec $\quad T(90000) = c\,(90000)^2 = 23$

150000 $\rightarrow$ $\quad T(150000) = c \cdot (150000)^2$

$$= 23 \times \left(\frac{150000}{90000}\right)^2$$

$$= 23 \times \left(\frac{5}{3}\right)^2 = 63\frac{8}{9}$$

$$\sim 64$$

Summations

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + \dots + f(b)$$

b $\rightarrow$ upper bound (int)

$i = a \rightarrow$ lower bound (int)

$\uparrow$ summation index

Short hand notation

```
int sum = 0;
for (int i = a; i <= b; i++)
    sum += f(i);
```

$$\sum_{i=3}^{7} (2^i + 3i) = (2^3 + 3\cdot3) + (2^4 + 3\cdot4) + (2^5 + 3\cdot5) + (2^6 + 3\cdot6) + (2^7 + 3\cdot7)$$

$$= (2^3 + 2^4 + 2^5 + 2^6 + 2^7) + (3\cdot3 + 3\cdot4 + 3\cdot5 + 3\cdot6 + 3\cdot7)$$

Rules

$$\sum_{i=a}^{b} (f(i) + g(i)) = \sum_{i=a}^{b} f(i) + \sum_{i=a}^{b} g(i)$$