Fall 2026 CIS 3362 Homework #5: Number Theory
Check WebCourses for the due date

1) (5 pts) Without the aid of a computer program, determine the prime factorization of 35,474,400. Show your work. You may do division on a calculator. State which numbers divided in evenly how many times.

2) (5 pts) What is φ(35,474,400)? You can use a calculator, but please show your work.

3) (10 pts) Use Fermat's Theorem to calculate the remainder when 276^{48436} is divided by 8363?

4) (10 pts) Use Euler's Theorem to calculate the remainder when $629^{10483201}$ is divided by 1435109?

5) (10 pts) Use the Fermat Factoring method by hand and calculator to factor the integer 31363859. Make a table showing your work. Your table should have exactly 10 rows.

6) (15 pts) For a prime number, p, under the assumption that there exists a primitive root, prove that there are exactly a total of $\phi(p-1)$ primitive roots.

7) (10 pts) Pick three prime numbers in between 20 and 50 and use a program to determine each of the primitive roots of the three primes you picked. State these primitive roots and verify that the number of primitive roots you found is exactly equal to of $\phi(p-1)$ in each case. Please attach your program separately and in your written answer write down each primitive root your program found.

8) (10 pts) $2^x \equiv 39 \ (mod\ 59)$. In class we went over a couple of ideas to speed up the computation of the value of x beyond just multiplying 2 by itself 59 times until we find the right-hand side. Use one of these ideas, showing your work by hand/calculator to determine the smallest positive integer value of x which satisfies this equation.

9) (25 pts) The prime number theorem says that the probability of an integer near the value of n being prime is roughly $\frac{1}{\ln n}$. Use the posted Miller-Rabin code and experimentally test this hypothesis. Generate random integers (say 10000 or more) in each of the following ranges:

(a) $10^{15} - 10^{16}$ (b) $10^{50} - 10^{51}$ (c) $10^{100} - 10^{101}$ (d) $10^{300} - 10^{301}$

Specifically, randomly generate 10,000 integers in this range, then use the Miller-Rabin test to see if those integers are prime or not. Compare your results to the values of $\frac{1}{\ln 10^{15}}$, $\frac{1}{\ln 10^{50}}$, $\frac{1}{\ln 10^{100}}$, and $\frac{1}{\ln 10^{300}}$. Are the results what you expected?