Resolving Motion Correspondence for Densely Moving Points

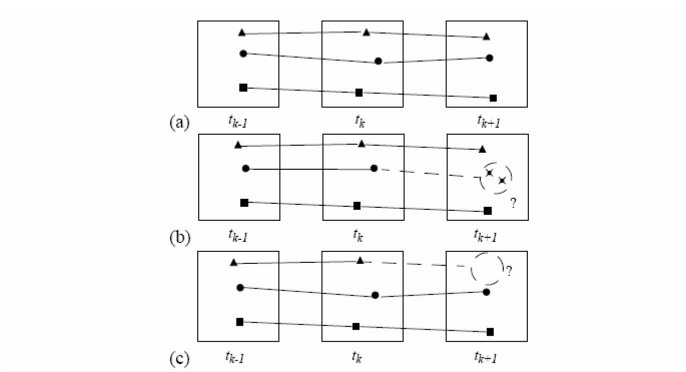
By C.J. Veenman,
M.J.T. Reinders,
and E. Backer

What are we going to do? Lecture plan

- ✓ Describe a task in general
- ✓ Describe a task more formally
- ✓ Discuss problems and constraints
- ✓ Discuss four algorithms to solve the task

General problem statement

Given a number of points in frame k, we are to find matching points in frame k + 1



Possible problems: false measurements, missing measurements, occlusions, points leaving and entering the frame, acceleration

Model constraints

We have to apply two important constraints:

- Individual points move smoothly from time instance to time instance.
- Total set of points move smoothly from time instance to time instance.

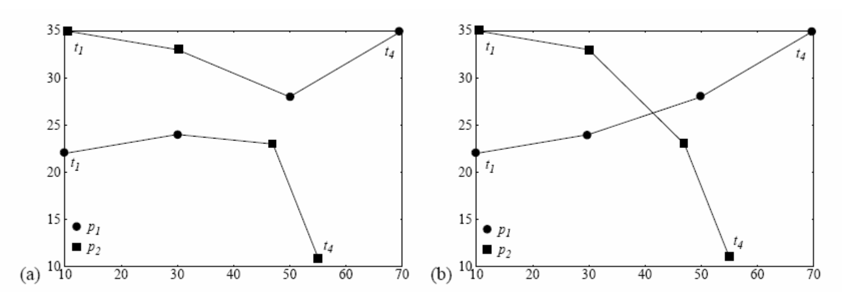
Based on these two constrains, two motion models are defined:

- *Individual motion model.*
- *Combined motion model.*

Formal problem definition

- There are M points, moving around in a 3-D world. We use index i to enumerate points
- There are n frames. Typically we denote current frame with letter k
- At each frame k there is a set X_k of m_k measurements $\mathbf{x}^k_j$ of point positions. Measurements are typically enumerated with index letter j
- The measurements $\mathbf{x}^k_j$ are vectors representing 2-D coordinates in a 2-D space, with dimensions Sw (width) and Sh (height). The number of measurements, m_k, can be either smaller (occlusion) or larger (false measurements) than M.
- At first two frames, all M points are identified among measurements.
- **The task is to return a set of M tracks that represent motion of the M points using the movements between first two frames as initial motion characteristics.**
- A point track T_i is an ordered list of corresponding measurements: $(\mathbf{x}^1_{j1,} \mathbf{x}^2_{j2} \ldots \mathbf{x}^n_{jn})$
- A track that has been formed up to frame k is called a track head and is denoted as T^k_i
- It is assumed that points do not enter or leave the scene

Tracks example for two points



Motion models: individual and combined motion

There are two kinds of motion models: *Individual motion model* and *Combined motion model*.

The *individual motion model* represents the motion of individual points. Acceleration information is not used thus the motion vector of a point can be estimated from only two consecutive measurements. On the basis of the motion vector and the individual motion model, the position of the point at the next time instance can be predicted. The measurement that is closest to this prediction can then be selected as corresponding measurement.

The *combined motion model* defines the motion smoothness constraint for the complete set of points. We use it to ensure that the average deviation from the individual motion models is minimal.

<u>The ‘costs’:</u>

Individual motion models introduce the notion of ‘cost’, that is, the cost c^k_{ij} of assigning measurement j to track i for frame k. The lower the cost, the better the match, so costs can be thought of as ‘deviations’ from the best prediction.

Individual motion models: smooth motion

- The smooth motion model introduced by Sethi and Jain assumes that the velocity magnitude and direction change gradually.
- The cost c^k_{ij} of assigning measurement j to track i can be evaluated by this formula:

$$c_{ij}^k = 0.1\left[1 - \frac{\left(\boldsymbol{x}_i^k - \boldsymbol{x}_{\alpha_i^k}^{k-1}\right)\cdot\left(\boldsymbol{x}_j^{k+1} - \boldsymbol{x}_i^k\right)}{\left\|\boldsymbol{x}_i^k - \boldsymbol{x}_{\alpha_i^k}^{k-1}\right\|\left\|\boldsymbol{x}_j^{k+1} - \boldsymbol{x}_i^k\right\|}\right] + 0.9\left[1 - 2\frac{\sqrt{\left\|\boldsymbol{x}_i^k - \boldsymbol{x}_{\alpha_i^k}^{k-1}\right\|\left\|\boldsymbol{x}_j^{k+1} - \boldsymbol{x}_i^k\right\|}}{\left\|\boldsymbol{x}_i^k - \boldsymbol{x}_{\alpha_i^k}^{k-1}\right\| + \left\|\boldsymbol{x}_j^{k+1} - \boldsymbol{x}_i^k\right\|}\right]$$

- The lower the cost, the better new measurement matches point motion. Ideally, cost is 0

Individual motion models: proximal uniformity

- The proximal uniformity model by Rangarajan and Shah assumes little motion in addition to constant speed.
- The cost c^k_{ij} of assigning measurement j to track i can be evaluated by this formula:

$$c_{ij}^{k} = \frac{\left\| \left(\boldsymbol{x}_i^k - \boldsymbol{x}_{\alpha_i^k}^{k-1} \right) - \left(\boldsymbol{x}_j^{k+1} - \boldsymbol{x}_i^k \right) \right\|}{\sum_{p=1}^{M} \sum_{q=1}^{m_{k+1}} \left\| \left(\boldsymbol{x}_p^k - \boldsymbol{x}_{\alpha_p^k}^{k-1} \right) - \left(\boldsymbol{x}_q^{k+1} - \boldsymbol{x}_p^k \right) \right\|} + \frac{\left\| \boldsymbol{x}_j^{k+1} - \boldsymbol{x}_i^k \right\|}{\sum_{p=1}^{M} \sum_{q=1}^{m_{k+1}} \left\| \boldsymbol{x}_q^{k+1} - \boldsymbol{x}_p^k \right\|},$$

- The lower the cost, the better new measurement matches point motion. Ideally, cost is 0

Combined motion models: average deviation

- This is a simple combined model which usually is realistic. It accounts for the average deviation from the optimal track according to the individual model
- Quantitatively, we use the generalized mean, which has a z parameter to differentiate between emphasis on large and small deviations from the optimal individual track

$$C^k(A^k, D^k) = \left[\frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{m_{k+1}} a_{ij}^k \left(c_{ij}^k \right)^z \right]^{\frac{1}{z}}$$

- Lower deviation C^k means better match between assigned measurements and point tracks.

Combined motion models: average deviation conditioned by competition and alternatives

- In this combined model measurements are assigned to that track head that gives low deviation from the optimal track, while both the other tracks are less attractive for this measurement and the other measurements are less attractive for this track.

$$C^k(A^k, D^k) = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{m_{k+1}}\left(a_{ij}^k c_{ij}^k - w_1 R_a(i) - w_2 R_c(j)\right)$$

where:

$$R_a(i) = \frac{1}{m_{k+1}-1}\sum_{q=1}^{m_{k+1}}\left(1-a_{iq}^k\right)c_{iq}^k; \qquad R_c(i) = \frac{1}{M-1}\sum_{p=1}^{M}\left(1-a_{pj}^k\right)c_{pj}^k$$

$R_a(i)$ represents the average cost of alternatives for T^k_i and R_c (j) the average cost for competitors of $\mathbf{x}^{k+1}_j$

Additional constraints

Some additional constraints can be used in algorithms:

- D_{max} and D_{min} – define the biggest and the lowest possible values for motion vector length
- S_{max} – defines the biggest possible deviation from individual motion model, imposes maximum possible cap on individual motion cost c^k_{ij}

Algorithms

We discuss the following algorithms:

- Salari and Sethi, PAMI, 1990
- Rangarajan and Shah, 1991
- Chetverikov and Verestoy, 1999
- Veenman, Reinders and Backer, PAMI, 2001

The goal of all algorithms is to assign measurements $\mathbf{x}^k_j$ to tracks i while minimizing combined motion C^k

Salari and Sethi algorithm, PAMI, 1990

- One of oldest algorithms.
  It uses smooth motion model and average deviation model

- For each new frame k:

  Step 1 *(initial)*. Assign nearest neighboring measurements to each track. Thus, to each track i, we assign measurement $\mathbf{x}^k_j$ most close to measurement $\mathbf{x}^{k-1}_i$ out of pool of unassigned measurements $\mathbf{x}^k$.
  Conflicts are not important on this step.

  Step 2 *(iterative)*. Compare all assignments against each other, calculate 'gain' function. If gain is positive, swap assigned measurements. Continue until no more swapping is done.
  Gain function for assignment pair (i, p) and (j, q) is defined as:

$$g^k_{ij} = c^k_{ip} + c^k_{jq} - (c^k_{iq} + c^k_{jp})$$

  So, if cost of assignments (i, p) and (j, q) is higher than cost of assignments (i, q) and (j, p) then we swap p and q. When no more swapping is possible, all costs are optimized.

Salari and Sethi algorithm, PAMI, 1990

The drawbacks:

- Iterative process is not guaranteed to converge, especially with densely moving points!
- If tracks are wrong at the start, exchanges in the remainder of the track will mostly be useless.
- Algorithm assumes a fixed number of points to be tracked and does not support occlusions or detection errors.
- If, at any frame k, we have less measurements than tracks, some tracks are lost. To solve this problem, phantom points are introduced that serve as replacements of missing measurements. By imposing the maximum allowed local smoothness criterion and a maximum speed, missed measurements are recognized and filled in with phantom points.

Rangarajan and Shah algorithm, 1991

- It uses the proximal uniformity individual motion model and conditioned combined motion model.
- This algorithm does not constrain the individual point motion. It does not have maximum speed constraint d_{max} or smoothness constraint s_{max} parameters.
- To assign points and to find the minimum of the combined motion model, non-iterative algorithm is used.

For each new measurement $\mathbf{x}^{k+1}_j$, for each track head T^k_i :

IF
on average all other track heads have a larger deviation with respect to $\mathbf{x}^{k+1}_j$
and
on average all other measurements have a worse criterion with respect to T^k_i
THEN
we assign measurement $\mathbf{x}^{k+1}_j$ to track T^k_i

This approach selects a measurement with reasonably low individual deviation from the optimal motion.

Rangarajan and Shah algorithm, 1991

The algorithm selects that assignment pair (i, j) that maximizes $R'_a(i) + R'_c(j)$ among all minimal track head extensions, where $R'_a(i)$ and $R'_c(j)$ are derived from definition for conditioned combined motion model:

$$R'_a(i) = \frac{1}{m_{k+1} - 1} \sum_{q=1, q \neq j}^{m_{k+1}} c^k_{iq}; \qquad R'_c(j) = \frac{1}{M - 1} \sum_{p=1, p \neq i}^{M} c^k_{pj}$$

Then, an optimal assignment pair $g(X_t, X_m)$ is repeatedly selected in the following way:

$$g(X_t, X_m) = \left((i, j) \mid i = \arg\max_{p \in X_t} \left(R'_a(p) + R'_c(j) \right), j = \arg\min_{q \in X_m} c^k_{pq} \right)$$

where X_t is the set of track head indices that have not yet been assigned a measurement, and X_m is the set of measurement indices that have not yet been assigned to a track head.

After an assignment has been found, the track head and measurement are removed from the respective index sets X_t and X_m.

The algorithm accumulates the assignment costs, and eventually stops when X_t is empty.

Rangarajan and Shah algorithm, 1991

Missing measurements:

Some measurements can be missing, by occlusion or otherwise.

In this case, because there is a lack of measurements at t_{k+1}, the problem is not which measurement should be assigned to which track head, but which track head should be assigned to which measurement.

When all track head assignments T^k_i to measurements $\mathbf{x}^k_j$ are found, it is clear for which tracks a measurement is missing. The algorithm directly fills in these points with extrapolated points.

Chetverikov and Verestoy, 1999

It uses the smooth individual motion model and conditioned combined motion model

1) The algorithm extends track heads T^k_i by first collecting all candidate measurements $\mathbf{x}^{k+1}_j$ in the circle with radius d_{max} around $\mathbf{x}^k_j$ whose criterion does not exceed s_{max}.

2) The candidate measurements are considered in optimal criterion order with respect to the track head.

3) Then, for each measurement all competing track heads are collected. The candidate measurement will be rejected if it is the best alternative for any of the competing track heads.

4) When there are no candidates left, the track head will not be connected. Remaining unconnected track parts, caused by occlusion or otherwise, are handled in a post-processing step,

Chetverikov and Verestoy, 1999

This algorithm can be summarized as follows:

Let $X_a(i)$ be the set of alternative track head extensions for track head T^k_i as defined below:

$$X_a(i) = \left\{ j \in X_m \mid i \in X_c(j), \forall p \in X_c(j) \left(j = \arg\min_{q \in X_m} c^k_{pq} \Longrightarrow p = i \right) \right\},$$

where each measurement $\mathbf{x}^{k+1}_j$ has a set of competing track heads X_c (j) according to:

$$X_c(j) = \left\{ i \in X_t \mid \left\| \boldsymbol{x}^{k+1}_j - \boldsymbol{x}^k_i \right\| < d_{max}, c^k_{ij} < \phi_{max} \right\}$$

The algorithm selects a measurement from $X_a(i)$ for a track head from X_t according to:

$$g(X_t, X_m) = \left((i, j) \mid i \in X_t, j = \arg\min_{q \in X_a(i)} c^k_{iq} \right)$$

This leads to the approximation of minimal combined motion criteria.

Veenman, Reinders and Backer, PAMI, 2001

Authors of this article proposed their own algorithm based on graph theory:

1) Costs c^k_{ij} are calculated for all combinations of points i and measurements j.

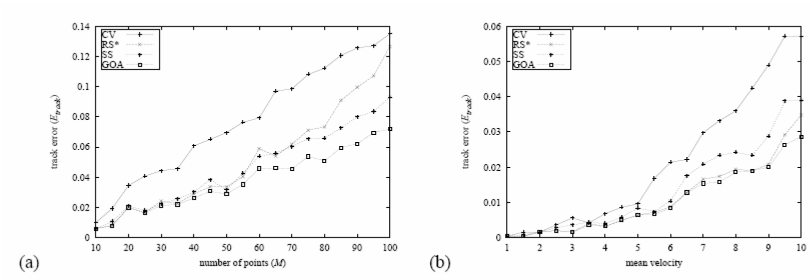
2) Then, bipartite weighted graph is constructed where points i represent the first set of vertices and measurements j represent the second

3) Costs c^k_{ij} are assigned as weights to all edges

4) Edges that violate maximum speed constraint d_{max} are removed

5) Then, Hungarian algorithm is applied to this graph that results in minimal cost assignment. All edges except resulting in minimal cost are pruned

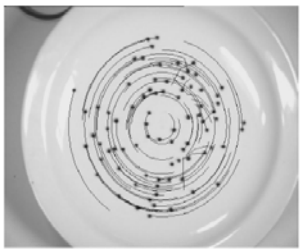
Performance graphs



(a) Track error as a function of increasingly dense point sets.

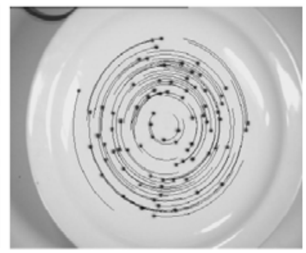
(b) Track error as a function of the mean velocity.

Experiment: Tracking seeds on a rotating dish



S&S: d_{max} = 42 p/s; 25 errors, 7.4 s

C&V: dmax = 42 p/s; 18 errors, 44 s



graph: dmax = 42 p/s; 0 errors, 90 ms