Lecture-8

Haralick's Edge Detector

Haralick's Edge Detector

- Fit a bi-quadratic polynomial to a small neighborhood of a pixel.
- Compute analytically second and third directional derivatives in the direction of gradient.
- If the second derivative is equal to zero, and the third derivative is negative, then that point is an edge point.

Haralick's Edge Detector

Bi-cubic polynomial:

$$f(x,y) = k_1 + k_2x + k_3y + k_4x^2 + k_5xy + k_6y^2 + k_7x^3 + k_8x^2y + k_9xy^2 + k_{10}y^3.$$

Gradient angle, defined with positive y-axis:

$$\sin\theta = \frac{k_2}{\sqrt{{k_2}^2 + {k_3}^2}}.$$

$$\cos\theta = \frac{k_3}{\sqrt{{k_2}^2 + {k_3}^2}}.$$

Homework

Directional derivative $f'_\theta = \frac{\partial f}{\partial x}\, cos\theta + \frac{\partial f}{\partial y} \sin\theta.$

Gradient angle, defined with positive x-axis:

Haralick's Edge Detector

$$f'_\theta = \frac{\partial f}{\partial x}\, cos\theta + \frac{\partial f}{\partial y} \sin\theta.$$

$$\begin{aligned} f'_\theta(x,y) &= \frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta, \\ f''_\theta(x,y) &= \frac{\partial^2 f}{\partial x^2}\sin^2\theta + \frac{\partial^2 f}{\partial y^2}\cos^2\theta + 2\frac{\partial^2 f}{\partial x \partial y}\cos\theta\sin\theta. \end{aligned}$$

Haralick's Edge Detector

$$f(x,y) = k_1 + k_2 x + k_3 y + k_4 x^2 + k_5 xy + k_6 y^2 + k_7 x^3 + k_8 x^2 y + k_9 xy^2 + k_{10} y^3.$$

$$x = \rho \sin\theta,\ y = \rho\cos\theta$$

$$f_\theta(\rho) = C_0 + C_1\rho + C_2\rho^2 + C_3\rho^3,$$

Homework

$$\begin{aligned}
C_0 &= k_1, \\
C_1 &= k_2 \sin\theta + k_3\cos\theta, \\
C_2 &= k_4 \sin^2\theta + k_5 \sin\theta\cos\theta + k_6\cos^2\theta, \\
C_3 &= k_7\sin^3\theta + k_8\sin^2\theta\cos\theta + k_9\sin\theta\cos^2\theta + k_{10}\cos^3\theta.
\end{aligned}$$

Haralick's Edge Detector

$$f_\theta(\rho) = C_0 + C_1\rho + C_2\rho^2 + C_3\rho^3,$$

$$\begin{aligned}
f_\theta'(\rho) &= C_1 + 2C_2\rho + 3C_3\rho^2, \\
f_\theta''(\rho) &= 2C_2 + 6C_3\rho, \\
f_\theta'''(\rho) &= 6C_3.
\end{aligned}$$

$f_\theta'''(\rho) < 0$, we get $6C_3 < 0$, or $C_3 < 0$.

$f_\theta''(\rho) = 2C_2 + 6C_3\rho = 0$, we get $\left|\frac{C_2}{3C_3}\right| < \rho_0$.

Haralick's Edge Detector

First order polynomial $f(x, y) = k_1 + k_2 x + k_3 y.$

9 points give 9 eqs

$$f1 = k_1 + k_2 x_1 + k_3 y_1,$$
$$f2 = k_1 + k_2 x_2 + k_3 y_2,$$
$$\vdots$$
$$f9 = k_1 + k_2 x_9 + k_3 y_9.$$

$$\begin{bmatrix} f1 \\ f2 \\ \vdots \\ f9 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & & \\ 1 & x_9 & y_9 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix},$$
$$f = Ak.$$

$$(A^T A)^{-1} A^T f = k,$$
$$Bf = k.$$

Haralick's Edge Detector

$B = (A^T A)^{-1} A^T$ is a 3×9 matrix

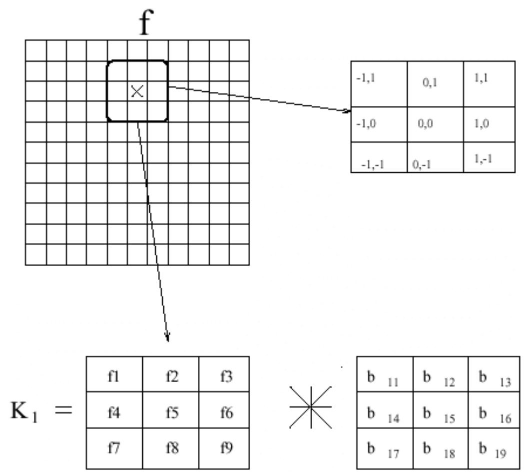
$$Bf = k.$$

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} & b_{17} & b_{18} & b_{19} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} & b_{27} & b_{28} & b_{29} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} & b_{37} & b_{38} & b_{39} \end{bmatrix} \begin{bmatrix} f1 \\ f2 \\ f3 \\ f4 \\ f5 \\ f6 \\ f7 \\ f8 \\ f9 \end{bmatrix} = \begin{bmatrix} k1 \\ k2 \\ k3 \end{bmatrix}$$

$$k_1 = b_{11} f1 + b_{12} f2 + b_{13} f3 + b_{14} f4 + b_{15} f5 + b_{16} f6 + b_{17} f7 + b_{18} f8 + b_{19} f9$$

$$k_1 = f * b_1.$$

Computing coefficients using convolution



$\frac{1}{175}$

-13	2	7	2	-13
2	17	22	17	2
7	22	27	22	7
2	17	22	17	2
-13	2	7	2	-13

k_1

$\frac{1}{420}$

31	-5	-17	-5	31
-44	-62	-68	-62	-44
0	0	0	0	0
44	62	68	62	44
-31	5	17	5	-31

k_2

$\frac{1}{420}$

31	-44	0	44	-31
-5	-62	0	62	5
-17	-68	0	68	17
-5	-62	0	62	5
31	-44	0	44	-31

k_3

$\frac{1}{70}$

2	2	2	2	2
-1	-1	-1	-1	-1
-2	-2	-2	-2	-2
-1	-1	-1	-1	-1
2	2	2	2	2

k_4

$\frac{1}{100}$

4	2	0	-2	-4
2	1	0	-1	-2
0	0	0	0	0
-2	-1	0	1	2
-4	-2	0	2	4

k_5

$\frac{1}{70}$

2	-1	-2	-1	2
2	-1	-2	-1	2
2	-1	-2	-1	2
2	-1	-2	-1	2
2	-1	-2	-1	2

k_6

$\frac{1}{60}$

-1	-1	-1	-1	-1
2	2	2	2	2
0	0	0	0	0
-2	-2	-2	-2	-2
1	1	1	1	1

k_7

$\frac{1}{140}$

-4	-2	0	2	4
2	1	0	-1	-2
4	2	0	-2	-4
2	1	0	-1	-2
-4	-2	0	2	4

k_8

$\frac{1}{140}$

-4	2	4	2	-4
-2	1	2	1	-2
0	0	0	0	0
2	-1	-2	-1	2
4	-2	-4	-2	4

k_9

$\frac{1}{60}$

-1	2	0	-2	1
-1	2	0	-2	1
-1	2	0	-2	1
-1	2	0	-2	1
-1	2	0	-2	1

k_{10}

Haralick's Edge Detector

1. Find $k_1, k_2, k_3, \ldots, k_{10}$ using least square fit, or masks given in Figure 2.8.
2. Compute θ, $\sin\theta$, $\cos\theta$.
3. Compute C_2, C_3.
4. If $C_3 < 0$ and $|\frac{C_2}{3C_3}| < \rho_0$ then that point is an edge point.

Figure 2.9: The steps in Haralick's Edge Detector.

Comparison of Three Edge Detectors

- Marr-Hildreth
  - Gaussian filter
  - Zerocrossings in Laplacian
- Canny
  - Gaussian filter
  - Maxima in gradient magnitude
- Haralick
  - Smoothing through bi-cubic polynomial
  - Zerocrossings in the second directional derivative, and negative third derivative

Laplacian and the second Directional Derivative and the direction of Gradient

$$\nabla^2 f = f_{xx} + f_{yy} = f_\theta'' + f_n''$$

$$f_\theta' = f_x \cos\theta + f_y \sin\theta$$

$$f_\theta'' = (f_{xx}\cos\theta + f_{yx}\sin\theta)\cos\theta + (f_{xy}\cos\theta + f_{yy}\sin\theta)\sin\theta$$

$$f_\theta'' = f_{xx}\cos^2\theta + f_{yy}\sin^2\theta + 2 f_{xy}\cos\theta\sin\theta$$

$$f_n'' = f_{xx}\cos^2 n + f_{yy}\sin^2 n + 2 f_{xy}\cos n \sin n$$

$$f_n'' = f_{xx}\cos^2(\theta + 90) + f_{yy}\sin^2(\theta + 90) + 2 f_{xy}\cos(\theta + 90)\sin(\theta + 90)$$

$$f_n'' = f_{xx}\sin^2\theta + f_{yy}\cos^2\theta - 2 f_{xy}\cos\theta\sin\theta$$

Laplacian and the second Directional Derivative and the direction of Gradient

$$f_\theta'' = f_{xx}\cos^2\theta + f_{yy}\sin^2\theta + 2 f_{xy}\cos\theta\sin\theta$$

$$f_n'' = f_{xx}\sin^2\theta + f_{yy}\cos^2\theta - 2 f_{xy}\cos\theta\sin\theta$$

$$\nabla^2 f = f_{xx} + f_{yy} = f_\theta'' + f_n''$$

Scales

- What should be sigma value for Canny and LG edge detection?
  - Marr-Hildreth:
- If use multiple sigma values (scales), how do you combine multiple edge maps?
  - *Spatial Coincidence* assumption:
    - Zerocrossings that coincide over several scales are physically significant.

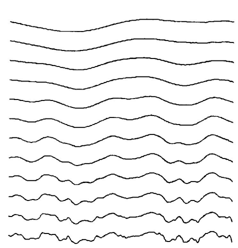
Scale Space

- Apply whole spectrum of scales
- Plot zerocrossings vs scales in a scale-space
- Interpret scale space contours
  - Contours are arches, open at the bottom, closed at the top
  - Interval tree
    - Each interval *I* corresponds to a node in a tree, whose parent node represents larger interval, from which interval *I* emerged, and whose off springs represents smaller intervals into which *I* subdivides.
    - Stability of a node is a scale range over which the interval exits.

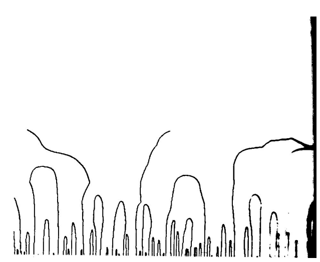
Scale Space

- Top level description
  - Iteratively remove nodes from the tree, splicing out nodes that are less stable than any of their parents and off springs

Scale Space

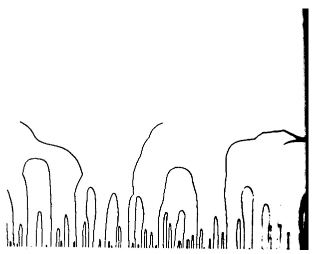


Multiple smooth versions of a signal

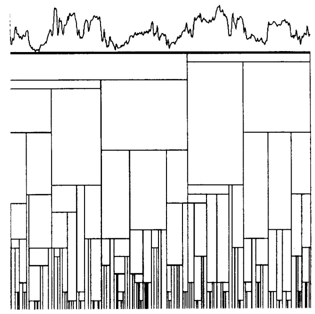


Zerocrossings at multiple scale

Scale Space



Scale Space



Interval Tree

Scale Space

A top level description of several signals using stability criterion.