Lecture

Computing Optical Flow

Horn&Schunck Optical Flow

$f(x, y, t)$ Image Sequence

$$\frac{df(x,y,t)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dx}{dt} + \frac{\partial f}{\partial t} = 0$$

brightness constancy eq

Horn&Schunck Optical Flow

$$f(x,y,t) = f(x+dx, y+dy, t+dt)$$

⇩ Taylor Series

$$f(x,y,t) = f(x,y,t) + \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$$

$$f_x dx + f_y dy + f_t dt = 0$$

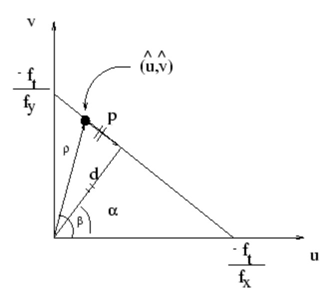
brightness constancy eq

Interpretation of optical flow eq

$$f_x u + f_y v + f_t = 0$$

$$v = -\frac{f_x}{f_y}u - \frac{f_t}{f_y}$$

Equation of st.line



d=normal flow

p=parallel flow

$$d = \frac{f_t}{\sqrt{f_x^2 + f_y^2}}$$

Horn&Schunck (contd)

$$\iint \{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\} dxdy$$

min

variational calculus

$$(f_x u + f_y v + f_t) f_x + \lambda(\nabla^2 u) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((\nabla^2 v) = 0$$

$$u = u_{av} - f_x \frac{P}{D}$$

$$v = v_{av} - f_y \frac{P}{D}$$

discrete version

$$(f_x u + f_y v + f_t) f_x + \lambda(u - u_{av}) = 0$$

$$(f_x u + f_y v + f_t) f_y + \lambda((v - v_{av}) = 0$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

$$\nabla^2 u = u_{xx} + u_{yy}$$

Algorithm-1

- k=0
- Initialize u^K v^K
- Repeat until some error measure is satisfied (converges)

$$u^K = u_{av}^{k-1} - f_x \frac{P}{D}$$

$$v = v_{av}^{K-1} - f_y \frac{P}{D}$$

$$P = f_x u_{av} + f_y v_{av} + f_t$$

$$D = \lambda + f_x^2 + f_y^2$$

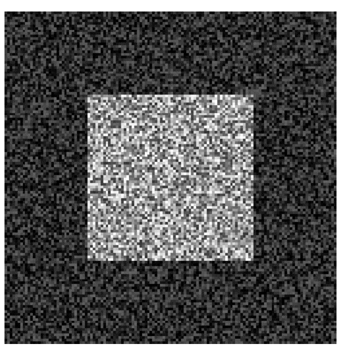
Derivative Masks

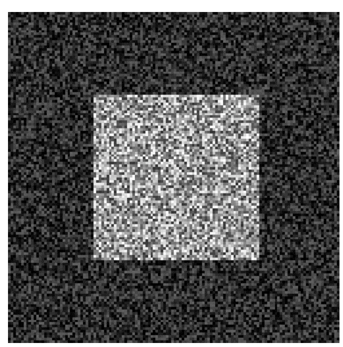
$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{first image} \qquad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \text{first image}$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \text{second image} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \text{second image} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \text{second image}$$

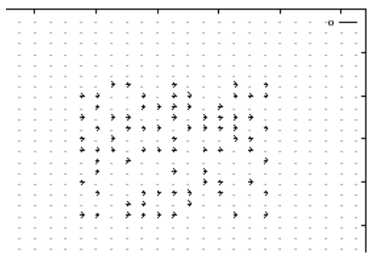
f_x $\qquad$ f_y $\qquad$ f_t

Synthetic Images

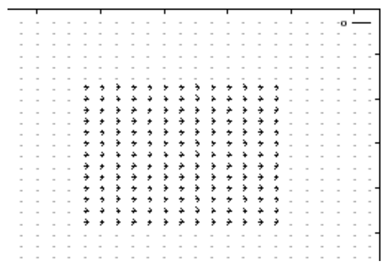




Results



One iteration



10 iterations

$\square = 4$