Lecture-11

Region Properties

Mid Term Exam

- March 8, Thursday
- Chapters 1, 2 & 3
- Closed book

Geometrical Properties

Area

$$A = \sum_{x=0}^{m} \sum_{y=0}^{n} B(x, y)$$

Centroid

$$\bar{x} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y)}{A}, \quad \bar{y} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)}{A}$$

Moments

General Moments

$$m_{pq} = \int \int x^p y^q B(x, y) dxdy$$

Discrete

$$M_x^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y), \; M_y^1 = \sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)$$

$$M_x^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} x^2 B(x, y), \; M_y^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} y^2 B(x, y), \; M_{xy}^2 = \sum_{x=0}^{m} \sum_{y=0}^{n} xyB(x, y)$$

Moments

Central Moments (Translation Invariant)

$$m_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q B(x,y) \; d(x-\bar{x})d(y-\bar{y})$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}} \qquad \text{Centroid}$$

Central Moments

$$m_{00} = m_{00} \equiv m$$
$$m_{01} = 0$$
$$m_{10} = 0$$
$$m_{20} = m_{20} - m\bar{x}^2$$
$$m_{11} = m_{11} - m\overline{xy}$$
$$m_{02} = m_{02} - m\bar{y}^2$$
$$m_{30} = m_{30} - 3m_{20}\bar{x} + 2m\bar{x}^3$$
$$m_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2m\bar{x}^2 y$$
$$m_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2m\bar{x}y^2$$
$$m_{03} = m_{03} - 3m_{02}\bar{y} + 2m\bar{y}^3$$

Moments

Hu Moments: translation, scaling and rotation invariant

$$u_1 = m_{20} + m_{02}$$

$$u2 = (m_{20} - m_{02})^2 + m_{11}{}^2$$

$$u_3 = (m_{30} - 3m_{12})^2 + (3m_{12} - m_{03})^2$$

$$u_4 = (m_{30} + m_{12})^2 + (m_{21} + m_{03})^2$$

$\vdots$

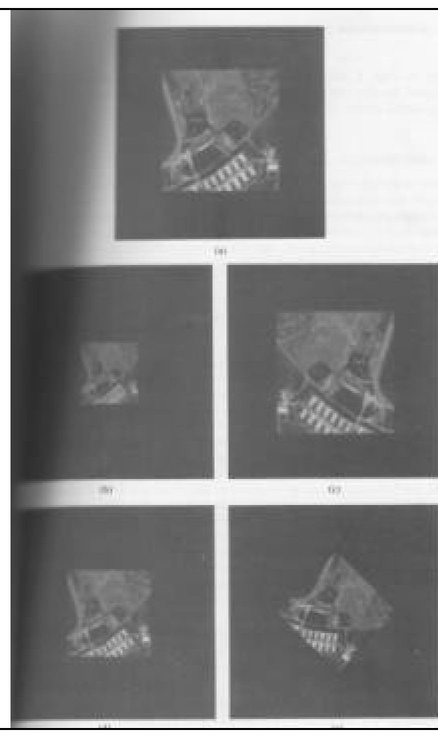


Table 8.2 Moment Invariants for the Images in Figs. 8.24(a)–(e)

Invariant (Log)	Original	Half Size	Mirrored	Rotated 2°	Rotated
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

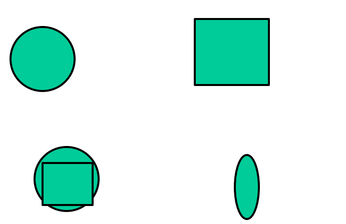
Hu moments

Perimeter & Compactness

Perimeter: The sum of its border points of the region. A pixel which has at least one pixel in its neighborhood from the background is called a border pixel.

Compactness

$$C = \frac{P^2}{4pA}$$



Orientation of the Region

Least second moment

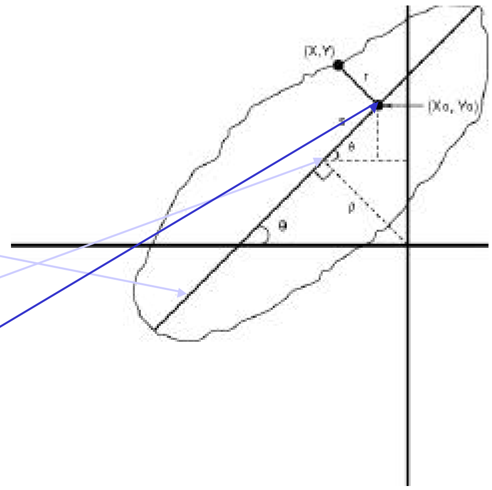
Minimize

$$E = \iint r^2 B(x, y)\,dxdy$$

$$x \sin \boldsymbol{q} - y \cos \boldsymbol{q} + \boldsymbol{r} = 0$$

$$(-r \sin q, r \cos q)$$

$$x_0 = -r \sin q + s \cos q$$
$$y_0 = r \cos q + s \sin q$$



Orientation of the Region

$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

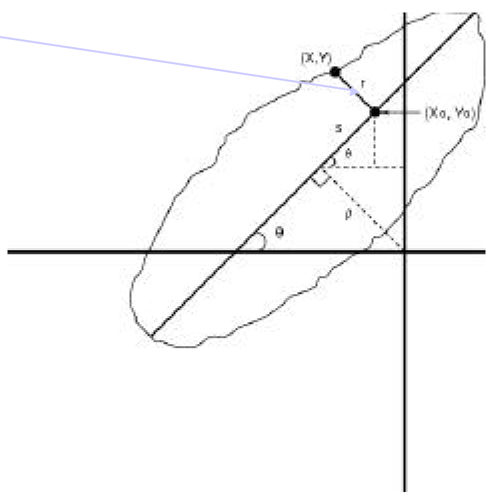
$$x_0 = -r \sin q + s \cos q$$
$$y_0 = r \cos q + s \sin q$$

Substituting (x_0,y_0) in r^2
And differentiating:

$$s = x \cos q + y \sin q$$

Substitute s in (x_0,y_0), then r:

$$r^2 = (x \sin q - y \cos q + r)^2$$



Orientation of the Region

$$r^2 = (x \sin q - y \cos q + r)^2$$

$$E = \iint r^2 B(x,y)\,dxdy$$

$$E = \iint (x \sin q - y \cos q + r)^2 B(x,y)\,dxdy$$

Substitute r in E and differentiate
Wrt to and equate it to zero

$$A(\bar{x} \sin q - \bar{y} \cos q + r) = 0$$

is the centroid

$$x' = x - \bar{x}, \quad y' = y - \bar{y}$$

$$E = a \sin^2 q - b \sin q \cos q + c \cos^2 q$$

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2q - \frac{1}{2} b \sin 2q$$

Substitute value of

$$a = \iint x'^2 B(x,y)\,dx'dy'$$

$$b = \iint x'y' B(x,y)\,dx'dy'$$

$$c = \iint y'^2 B(x,y)\,dx'dy'$$

Orientation of the Region

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2q - \frac{1}{2} b \sin 2q$$

Differentiating this wrt

$$\tan 2q = \frac{b}{a-c}$$

$$\sin 2q = \pm \frac{b}{\sqrt{b^2 + (a-c)^2}}$$

$$\cos 2q = \pm \frac{a-c}{\sqrt{b^2 + (a-c)^2}}$$

$$a = \iint x'^2 B(x,y)\,dx'dy'$$

$$b = \iint x'y' B(x,y)\,dx'dy'$$

$$c = \iint y'^2 B(x,y)\,dx'dy'$$

$$x' = x - \bar{x}, \quad y' = y - \bar{y}$$

$$a = \sum \sum x^2 B(x,y) - A\bar{x}^2$$

$$b = 2 \sum \sum xyB(x,y) - A\overline{xy}$$

$$c = \sum \sum y^2 B(x,y) - A\bar{y}^2$$

Example

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find: area, centroid, moments, compactness, perimeter, orientation