CAP 5415 Computer Vision
Fall 2005

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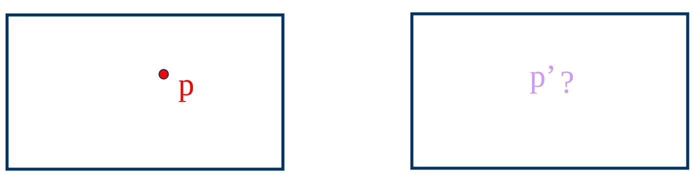
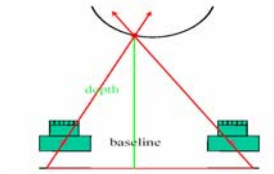
Office: CSB 250

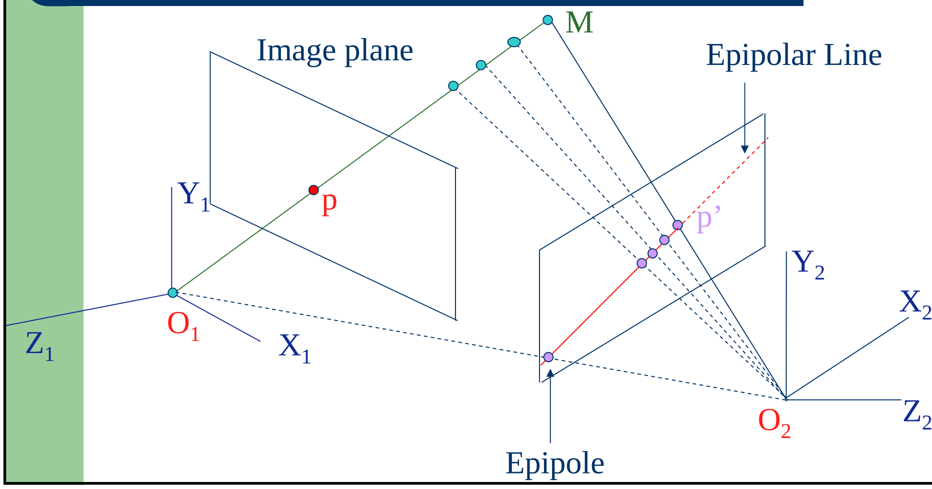
Stereo Constraints



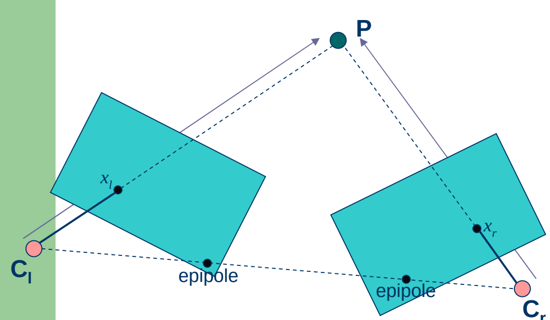
Given p in left image, where can the corresponding point p' in right image be?

Stereo Constraints
Epipolar Geometry





Basics



$T = C_l C_r$

$P_l = P C_l$

$P_r = R(P_l - T)$

- Co-planarity condition
  - 3 vectors on a plane

$$A^T (B \times C) = 0$$

  - Let's define for epipolar plane

$$(P_l - T)^T (T \times P_l) = 0$$

$$(R^T P_r)^T (T \times P_l) = 0$$

$$P_r^T RSP_l = 0$$

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Essential Matrix

- Related to camera extrinsic parameters
  - Rotation and translation

$$P_r^T RSP_l = 0$$

$$E = RS$$

- Captures relation between to camera coordinates

How do we relate image coordinates?

- From camera coordinates to image coordinates.
  - Intrinsic camera parameters (in homogenous coordinates)

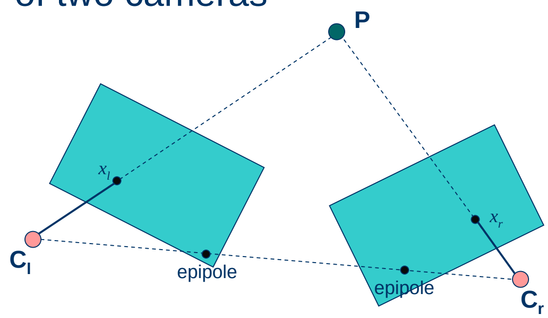
$$x_r^T M_r^{-T} RSM_t^{-1} x_l = 0$$

$$E = RS$$

$$\left.\begin{aligned} F &= M_r^{-T} RSM_t^{-1} \\ F &= M_r^{-T} EM_t^{-1} \end{aligned}\right\} \text{Fundamental matrix}$$

Fundamental Matrix

- Defines relation between two image planes of two cameras



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Fundamental Matrix

- Relates left image coordinates to right image coordinates.
- Includes all camera parameters
- 3x3 matrix with 7 degrees of freedom
  - 4 for epipoles (x,y) and 3 for point projection
- Rank 2 (due to rank deficient S)

$$\begin{bmatrix} x & y & 1 \end{bmatrix} F \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = 0$$

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$$[x \quad y \quad 1] F \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = 0$$

Computing Fundamental Matrix

- Eight point algorithm
  - Most common approach
  - Requires 8 corresponding points in both images
- Write unknown fundamental matrix parameters ($f_1..f_9$) into vector and the rest into equation (observation) matrix **O**.

  $$\mathbf{O}.f = 0$$
- Compute least squares solution

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Computing Fundamental Matrix 8-Point Algorithm

- Requires 8 corresponding points in both images
- Write unknown fundamental matrix parameters (f1..f9) into vector and the rest into equation (observation) matrix.

$$[x \quad y \quad 1] \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = 0$$

$$[xx' \quad xy' \quad x \quad yx' \quad yy' \quad y \quad x' \quad y' \quad 1] f = 0$$

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix}$$

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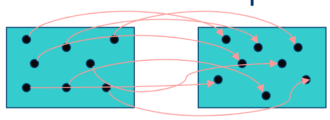
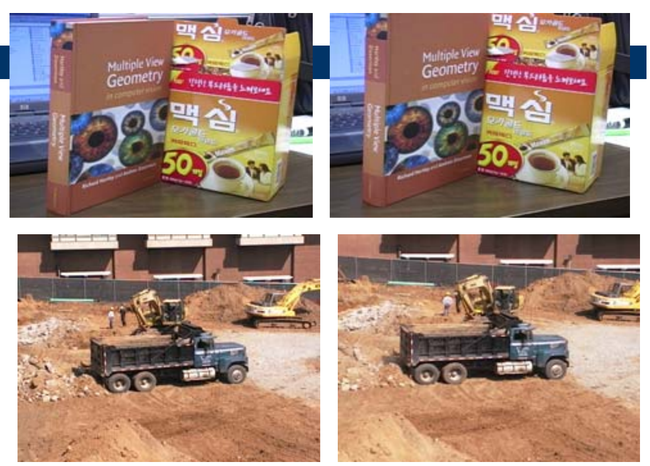
8-Point Algorithm

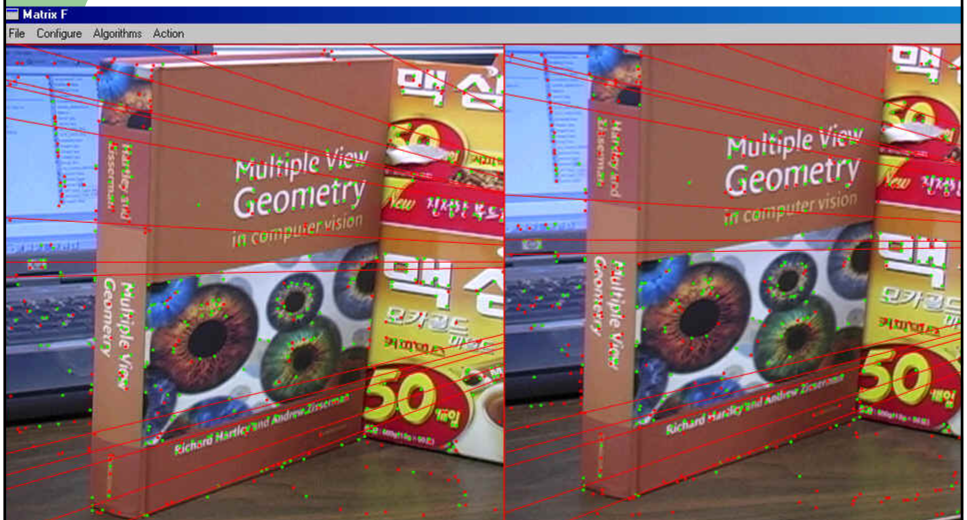
- Let there be N corresponding points in both images



$$\begin{bmatrix} x_1x_1' & x_1y_1' & x_1 & y_1x_1' & y_1y_1' & y_1 & x_1' & y_1' & 1 \\ \vdots & & & & & & & & \\ x_Nx_N' & x_Ny_N' & x_N & y_Nx_N' & y_Ny_N' & y_N & x_N' & y_N' & 1 \end{bmatrix} f = 0$$

$$Of = 0$$





$$Of = 0$$

8-Point Algorithm

- Due to 1s in the last column of O rank of O is 8

$$O^T Of = 0$$

- Homogenous equation, solution is given using SVD (or eigenspace decomposition)
- Perform eigenspace decomposition and select minimum eigenvalued eigenvector as solution
- Let solution be e_{min}

8-Point Algorithm

- e_{min} should satisfy rank 2 constraint.

$$e_{\min} = [e_1 \quad e_2 \quad e_3 \quad e_4 \quad e_5 \quad e_6 \quad e_7 \quad e_8 \quad e_9]$$

$$f = [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6 \quad f_7 \quad f_8 \quad f_9]$$

$$F = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \qquad E_{\min} = \begin{bmatrix} e_1 & e_2 & e_3 \\ e_4 & e_5 & e_6 \\ e_7 & e_8 & e_9 \end{bmatrix}$$

- Perform SVD on E_{min}.

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Enforcing Rank Constraint

$$E_{\min} = \begin{bmatrix} e_1 & e_2 & e_3 \\ e_4 & e_5 & e_6 \\ e_7 & e_8 & e_9 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 \\ u_4 & u_5 & u_6 \\ u_7 & u_8 & u_9 \end{bmatrix} \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_3 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \\ v_4 & v_5 & v_6 \\ v_7 & v_8 & v_9 \end{bmatrix}$$

- Set s_3 to 0
- Recompute E_{min} which should be your estimated fundamental matrix

$$\hat{F} = \begin{bmatrix} u_1 & u_2 & u_3 \\ u_4 & u_5 & u_6 \\ u_7 & u_8 & u_9 \end{bmatrix} \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 & v_2 & v_3 \\ v_4 & v_5 & v_6 \\ v_7 & v_8 & v_9 \end{bmatrix}$$

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Scaling and Normalization in 8-Point Algorithm

- It is important to normalize and scale the input points (tokens on images)
- Perform separately for both images
  - Take mean of x and y values, μ_x and μ_y.
  - Subtract mean from every point (translate origin to mean)
  - Divide x and y of each point by $\mu_x/2$ and $\mu_y/2$ (on the average each point is (1,1)
- Construct the observation matrix
- Compute fundamental matrix

Suggested Reading

- Emanuele Trucco, Alessandro Verri, "Introductory Techniques for 3-D Computer Vision", Prentice Hall, 1998
  - Section 7.3