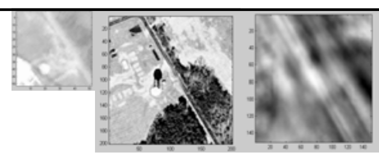
CAP 5415 Computer Vision
Fall 2005

Dr. Alper Yilmaz

Univ. of Central Florida

www.cs.ucf.edu/courses/cap5415/fall2005

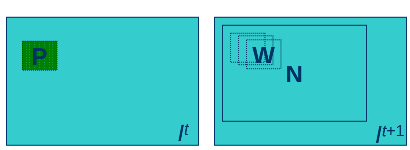
Office: CSB 250



Recap
Block Based Optical Flow

- Select a patch **P** image at time t
- Search for **P** at frame t+1 in a larger neighborhood **N**
  - Overlapping windows **W** in **N**
  - Compute similarity between **W** and **P**
  - Select the location with highest similarity

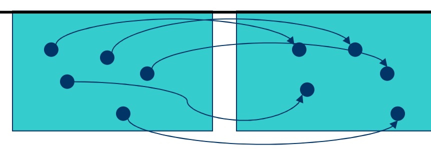
SSD
AD
CC
NC
MC



Recap
Issues With Correlation

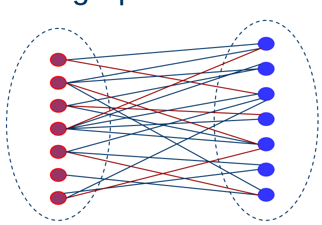
- Patch Size
- Search Area
- How many peaks
- Computationally expensive
  - Same operations in Fourier domain takes less time
    - Take FFT of image patch and search area
    - Multiply Fourier coefficients to construct corr. surface
    - Find maximum
- Should use pyramids here too for large displacements

Alper Yilmaz, Fall 2005 UCF



Recap
Token Based Optical Flow

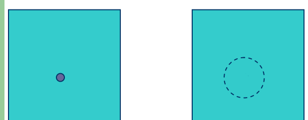
- Interest points, corners, edges
- Find tokens in both (multiple) frames
- Correspondence provides optical flow
  - Represent tokens and possible correspondences in graph



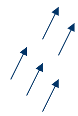
- Every correspondence has weight
- Find max. matching
  - Sum of correspondences is max.

Alper Yilmaz, Fall 2005 UCF

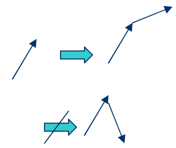
Recap
Correspondence Weights



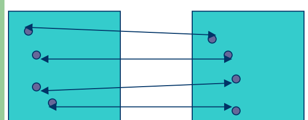
Maximum Velocity



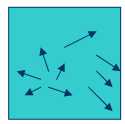
Common Motion



Minimum Velocity



Consistent Match



Model

Recap
Weights

geometric mean $GM(a,b)=\sqrt{ab}$

aritmetic mean $AM(a,b)=\frac{a+b}{2}$

Ullman: $w_{ij}=\|y_j-x_i\|$

(favors closest points)

Sethi & Jain: $w_{ij}=c\left[1-\frac{(y_j-x_i).v_{x_i}}{\|y_j-x_i\|\|v_{x_i}\|}\right]+(1-c)\left[1-\frac{GM(\|y_j-x_i\|,\|v_{x_i}\|)}{AM(\|y_j-x_i\|,\|v_{x_i}\|)}\right]$

(favors points with similar velocities)

cosine of angle between vectors
Favors similar velocities

Rangarajan & Shah: $w_{ij}=\frac{\|v_{x_i}-(y_j-x_i)\|}{\sum_{k=1}^{p}\sum_{l=1}^{q}\|v_{x_k}-(y_l-x_k)\|}+\frac{\|y_j-x_i\|}{\sum_{k=1}^{p}\sum_{l=1}^{q}\|y_l-x_p\|}$

(favors points with similar velocities)

normalization

Structure From Motion

Alper Yilmaz, Fall 2005 UCF

Definition

- Given optical flow or point correspondences, compute 3-D motion and depth.
  - What projection?
  - How many points?
  - How many frames?
- Non-linear problem.

Alper Yilmaz, Fall 2005 UCF

3D Motion

- Displacement model
- Velocity model

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \mathbf{T} = \underbrace{\begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix}}_{\text{Rotation matrix using Euler angles}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} \Rightarrow \begin{aligned} X' &= X - \alpha Y + \beta Z + T_X \\ Y' &= \alpha X + Y - \gamma Z + T_Y \\ Z' &= -\beta X + \gamma Y + Z + T_Z \end{aligned}$$

Rotation matrix
using Euler angles

Alper Yilmaz, Fall 2005 UCF

Displacement Model and Its Projection onto Image Space

$$X' = X - \alpha Y + \beta Z + T_X$$
$$Y' = \alpha X + Y - \gamma Z + T_Y$$
$$Z' = -\beta X + \gamma Y + Z + T_Z$$

Orthographic projection

$$x' = x - \alpha y + \beta Z + T_X$$
$$y' = \alpha x + y - \gamma Z + T_Y$$

Perspective projection

$$x' = \frac{X - \alpha Y + \beta Z + T_X}{-\beta X + \gamma Y + Z + T_Z} \qquad x' = \frac{x - \alpha y + \beta + \frac{T_X}{Z}}{-\beta x + \gamma y + 1 + \frac{T_Z}{Z}}$$

$$y' = \frac{\alpha X + Y - \gamma Z + T_Y}{-\beta X + \gamma Y + Z + T_Z} \qquad y' = \frac{\alpha x + y - \gamma + \frac{T_Y}{Z}}{-\beta x + \gamma y + 1 + \frac{T_Z}{Z}}$$

Alper Yilmaz, Fall 2005 UCF

Velocity Model in 3D Optical Flow

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Starting with the displacement

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \left(\begin{bmatrix} 0 & -\alpha & \beta \\ \alpha & 0 & -\gamma \\ -\beta & \gamma & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} X'-X \\ Y'-Y \\ Z'-Z \end{bmatrix} = \begin{bmatrix} 0 & -\alpha & \beta \\ \alpha & 0 & -\gamma \\ -\beta & \gamma & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

Alper Yilmaz, Fall 2005 UCF

Velocity Model in 3D Optical Flow

$$\begin{bmatrix} X'-X \\ Y'-Y \\ Z'-Z \end{bmatrix} = \begin{bmatrix} 0 & -\alpha & \beta \\ \alpha & 0 & -\gamma \\ -\beta & \gamma & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

rotational velocities

translational velocities

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix}$$

3D optical flow

$$\dot{X} = \Omega_2 Z - \Omega_3 Y + V_1$$
$$\dot{Y} = \Omega_3 X - \Omega_1 Z + V_2$$
$$\dot{Z} = \Omega_1 Y - \Omega_2 X + V_3$$

$$\Rightarrow \quad \dot{\mathbf{X}} = \Omega \times \mathbf{X} + \mathbf{V}$$

cross product

Alper Yilmaz, Fall 2005 UCF

$$\dot{X} = \Omega_2 Z - \Omega_3 Y + V_1$$
$$\dot{Y} = \Omega_3 X - \Omega_1 Z + V_2$$
$$\dot{Z} = \Omega_1 Y - \Omega_2 X + V_3$$

Velocity Model in 2D

- Orthographic projection

$$x = X, \quad y = Y \;\Rightarrow\; u = \dot{x} = \dot{X}, \quad v = \dot{y} = \dot{Y} \;\Rightarrow\; u = \Omega_2 Z - \Omega_3 y + V_1, \quad v = \Omega_3 x - \Omega_1 Z + V_2$$

- Perspective projection

$$x = f\frac{X}{Z}, \quad y = f\frac{Y}{Z} \;\Rightarrow\; u = \dot{x}, \quad v = \dot{y} \;\Rightarrow$$
$$u = f(\frac{V_1}{Z} + \Omega_2) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$
$$v = f(\frac{V_2}{Z} - \Omega_1) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

HOMEWORK

$$u = f(\frac{V_1}{Z} + \Omega_2) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$
$$v = f(\frac{V_2}{Z} - \Omega_1) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

Pure Translation Perspective Motion Model

$$u = \frac{fV_1 - V_3 x}{Z} + f\Omega_2 - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$
$$v = \frac{fV_2 - V_3 y}{Z} - f\Omega_1 + \Omega_3 x + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

rotational

translational

Pure Translation

- p_0 is the vanishing point of the direction of translation.
- p_0 is the intersection of the ray parallel to the translation vector with the image plane.

$$u = \frac{fV_1}{Z} - \frac{V_3}{Z}x \qquad v = \frac{fV_2}{Z} - \frac{V_3}{Z}y$$

Let

$$x_0 = f\frac{V_1}{V_3} \qquad y_0 = f\frac{V_2}{V_3}$$

$$\Rightarrow \quad u = (x_0 - x)\frac{V_3}{Z} \qquad v = (y_0 - y)\frac{V_3}{Z}$$

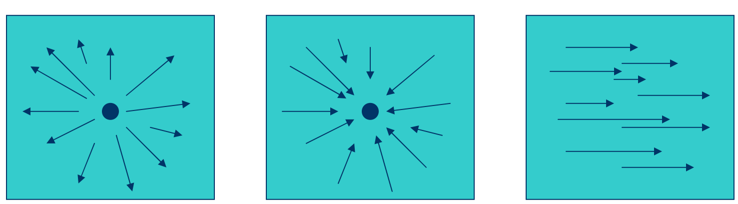
$$p_0 = (x_0, y_0)$$

$$u = (x_0 - x)\frac{V_3}{Z} \qquad v = (y_0 - y)\frac{V_3}{Z}$$

$$p_0 = \left(f\frac{V_1}{V_3}, f\frac{V_2}{V_3} \right)$$

Pure Translation

- If $V_3 \neq 0$, the flow field is radial, and all vectors point towards (or away from) a single point.
- If $V_3 = 0$, the flow field is parallel.



Structure From Motion

Orthographic projection

Alper Yilmaz, Fall 2005 UCF

$$x' = x - \alpha y + \beta Z + T_X$$
$$y' = \alpha x + y - \gamma Z + T_Y$$

SFM Orthographic *Displacement*

- Two step simple approach
  - Assume depth *Z* is known compute motion parameters α, β, γ, T_x, T_y
  - Using new motion parameters refine depth *Z*

$$\begin{bmatrix} x'-x \\ y'-y \end{bmatrix} = \begin{bmatrix} -y & Z & 0 & 1 & 0 \\ x & 0 & -Z & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ T_x \\ T_y \end{bmatrix}$$

Step 1

$$\begin{bmatrix} \beta \\ -\gamma \end{bmatrix} [Z] = \begin{bmatrix} x'-x-\alpha y - T_x \\ y'-y-\alpha x - T_y \end{bmatrix}$$

Step 2

Alper Yilmaz, Fall 2005 UCF

SFM
Perspective Projection

- Find translation **V** using search.
- Find rotation Ω using least squares fit.
- Find Depth **Z**.

Alper Yilmaz, Fall 2005 UCF

$$u = -\left(\frac{V_1}{Z} + \Omega_2\right) + \frac{V_3}{Z}x + \Omega_3 y + \Omega_1 xy - \Omega_2 x^2$$

$$v = -\left(\frac{V_2}{Z} - \Omega_1\right) - \Omega_3 x + \frac{V_3}{Z}y - \Omega_2 xy + \Omega_1 y^2$$

Heeger&Jepson Method

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} = \frac{1}{Z(x,y)} \begin{bmatrix} -1 & 0 & x \\ 0 & -1 & y \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} xy & -(1+x^2) & y \\ 1+y^2 & -xy & -x \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix}$$

$$\Theta(\mathbf{x},\mathbf{y}) = p(x,y)\mathbf{A}(x,y)\mathbf{V} + \mathbf{B}(x,y)\Omega$$

This is for one point. We have multiples of points so we will form a system of equations.

Alper Yilmaz, Fall 2005 UCF

$$\Theta(\mathbf{x},\mathbf{y}) = p(x,y)\mathbf{A}(x,y)\mathbf{V} + \mathbf{B}(x,y)\Omega$$

Heeger&Jepson Method

$$\begin{bmatrix}\Theta(x_1,y_1)\\ \vdots\\ \vdots\\ \vdots\\ \Theta(x_n,y_n)\end{bmatrix} = \begin{bmatrix}\mathbf{A}(x_1,y_1)\mathbf{V} & 0 & 0 & \dots & 0\\ 0 & & & & 0\\ 0 & & & & 0\\ \vdots & & & & \vdots\\ \mathbf{0} & 0 & 0 & \dots & \mathbf{A}(x_n,y_n)\mathbf{V}\end{bmatrix}\begin{bmatrix}p(x_1,y_1)\\ p(x_2,y_2)\\ p(x_3,y_3)\\ \vdots\\ p(x_n,y_n)\end{bmatrix} + \begin{bmatrix}\mathbf{B}(x_1,y_1)\\ \mathbf{B}(x_2,y_2)\\ \mathbf{B}(x_3,y_3)\\ \vdots\\ \mathbf{B}(x_n,y_n)\end{bmatrix}\Omega$$

$$\begin{bmatrix}\Theta(x_1,y_1)\\ \vdots\\ \vdots\\ \vdots\\ \Theta(x_n,y_n)\end{bmatrix} = \begin{bmatrix}\mathbf{A}(x_1,y_1)\mathbf{V} & 0 & 0 & \dots & 0 & \mathbf{B}(x_1,y_1)\\ 0 & & & & & \mathbf{B}(x_2,y_2)\\ 0 & & & & & \mathbf{B}(x_3,y_3)\\ \vdots & & & & & \vdots\\ \mathbf{0} & 0 & 0 & \dots & \mathbf{A}(x_n,y_n)\mathbf{V} & \mathbf{B}(x_n,y_n)\end{bmatrix}\begin{bmatrix}p(x_1,y_1)\\ p(x_2,y_2)\\ \vdots\\ p(x_n,y_n)\\ \Omega\end{bmatrix}$$

$$\Theta = \mathbf{C}(\mathbf{V})\mathbf{q}$$

Alper Yilmaz, Fall 2005 UCF

$$\Theta = \mathbf{C}(\mathbf{V})\mathbf{q}$$

Find Translation Using Search

- Define energy functional

$$E(\mathbf{V},\mathbf{q}) = \left\|\Theta - \mathbf{C}(\mathbf{V})\mathbf{q}\right\|^2$$

- Decompose **C(V)** using QR decomposition

$$\mathbf{C}(\mathbf{V}) = \overline{\mathbf{C}}(\mathbf{V})\mathbf{U}(\mathbf{V})$$

Orthonormal matrix

Upper triangular matrix

$$E(\mathbf{V},\mathbf{q}) = \left\|\Theta - \overline{\mathbf{C}}(\mathbf{V})\mathbf{U}(\mathbf{V})\mathbf{q}\right\|^2$$

$$E(\mathbf{V},\mathbf{q}) = \left\|\Theta - \overline{\mathbf{C}}(\mathbf{V})\overline{\mathbf{q}}\right\|^2$$

Alper Yilmaz, Fall 2005 UCF

$$E(\mathbf{V},\mathbf{q}) = \left\|\Theta - \overline{\mathbf{C}}(\mathbf{V})\overline{\mathbf{q}}\right\|^2$$

Find Translation Using Search

- Take derivative and replace q

$$\overline{\mathbf{C}}^T(\mathbf{V})\left(\Theta - \overline{\mathbf{C}}(\mathbf{V})\overline{\mathbf{q}}\right) = 0$$

Due to orthogonality becomes identity

$$\overline{\mathbf{C}}^T(\mathbf{V})\Theta - \overline{\mathbf{C}}^T(\mathbf{V})\overline{\mathbf{C}}(\mathbf{V})\overline{\mathbf{q}} = 0 \quad \Rightarrow \quad \overline{\mathbf{q}} = \overline{\mathbf{C}}^T(\mathbf{V})\Theta$$

$$E(\mathbf{V}) = \left\|\Theta - \overline{\mathbf{C}}(\mathbf{V})\overline{\mathbf{C}}^T(\mathbf{V})\Theta\right\|^2$$

$$E(\mathbf{V}) = \left\|\left(I - \overline{\mathbf{C}}(\mathbf{V})\overline{\mathbf{C}}^T(\mathbf{V})\right)\Theta\right\|^2$$

Alper Yilmaz, Fall 2005 UCF

$$E(\mathbf{V}) = \left\|\left(I - \overline{\mathbf{C}}^T(\mathbf{V})\overline{\mathbf{C}}(\mathbf{V})\right)\Theta\right\|^2$$

$$\Theta(\mathbf{x},\mathbf{y}) = p(x,y)\mathbf{A}(x,y)\mathbf{V} + \mathbf{B}(x,y)\Omega$$

Find Translation Using Search

- Define the orthonormal basis for orthogonal complement of **C(V)** $\mathbf{C}^{\perp}(\mathbf{V})$

$$E(\mathbf{V}) = \left\|\Theta^T\mathbf{C}^{\perp}(V)\right\|^2$$

- Minimize this over all possible translations
  - Define possible **V**
  - Compute **A** and **B**
  - See which minimizes this error

Alper Yilmaz, Fall 2005 UCF

$$\Theta(\mathbf{x},\mathbf{y}) = p(x,y)\mathbf{A}(x,y)\mathbf{V} + \mathbf{B}(x,y)\Omega$$

Find Rotation

- Let vector **d** be perpendicular to *p***A**
- Multiply all parts of equation with **d**

$$\overbrace{d^T(x,y,V)\Theta(\mathbf{x},\mathbf{y})}^{D} = \overbrace{d^T(x,y,V)\mathbf{B}(x,y)}^{E}\Omega$$

$$\Omega = \left(E^T E\right)^{-1} E^T D$$

Alper Yilmaz, Fall 2005 UCF

Find depth

- Find depth for each pixel (x,y) using

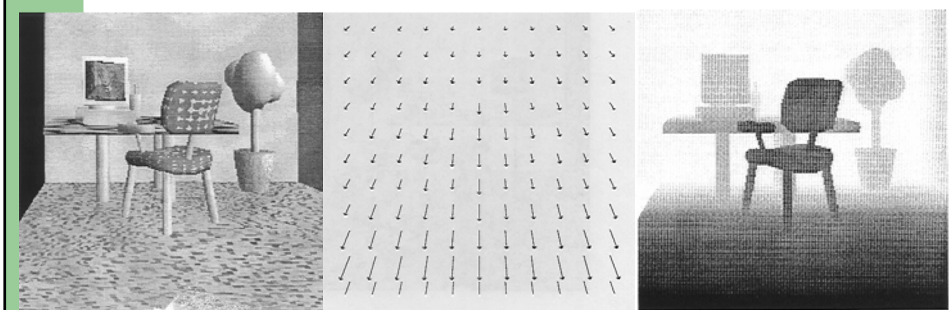
$$u = -(\frac{V_1}{Z} + \Omega_2) + \frac{V_3}{Z}x + \Omega_3 y + \Omega_1 xy - \Omega_2 x^2$$

$$v = -(\frac{V_2}{Z} - \Omega_1) - \Omega_3 x + \frac{V_3}{Z}y - \Omega_2 xy + \Omega_1 y^2$$

Alper Yilmaz, Fall 2005 UCF

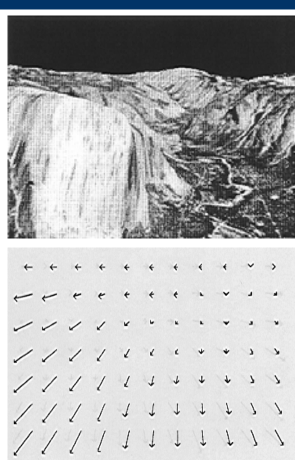
Example

- Synthetic image

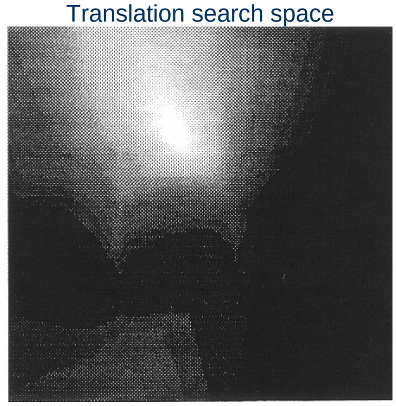


Alper Yilmaz, Fall 2005 UCF

Example



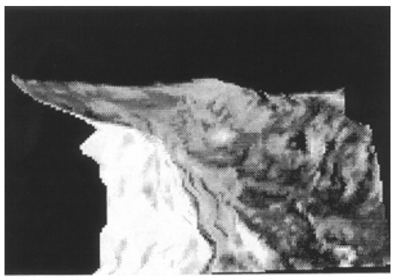
Translation search space

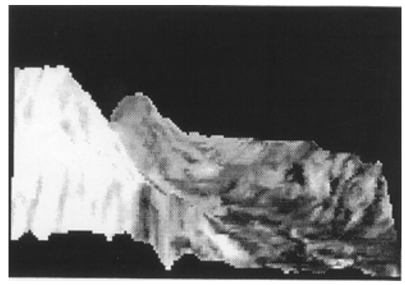


Alper Yilmaz, Fall 2005 UCF

Example

- Two views of reconstructed scene





Alper Yilmaz, Fall 2005 UCF

Homework (Due date 9 Nov. 2005)

- Given

$$x = f\frac{X}{Z} \qquad y = f\frac{Y}{Z}$$

$$u = \dot{x} \qquad v = \dot{y}$$

$$\dot{X} = \Omega_2 Z - \Omega_3 Y + V_1$$
$$\dot{Y} = \Omega_3 X - \Omega_1 Z + V_2$$
$$\dot{Z} = \Omega_1 Y - \Omega_2 X + V_3$$

- Derive

$$u = f(\frac{V_1}{Z} + \Omega_2) - \frac{V_3}{Z}x - \Omega_3 y - \frac{\Omega_1}{f}xy + \frac{\Omega_2}{f}x^2$$

$$v = f(\frac{V_2}{Z} - \Omega_1) + \Omega_3 x - \frac{V_3}{Z}y + \frac{\Omega_2}{f}xy - \frac{\Omega_1}{f}y^2$$

Alper Yilmaz, Fall 2005 UCF

Programming Project #2
Due date 5 December 2005

- Implement Anandan's approach for Global motion compensation.
  1. Build Gaussian pyramid
  2. Start from lowest resolution compute affine parameters iteratively
  3. Go to next pyramid level by projecting affine parameters
     1. You need to multiple translation of the affine parameters by 2
  4. Display warped images
- Deliverables
  – Report including all **pyramid level images and warped images** of each pyramid level along with softcopy **source code**

Alper Yilmaz, Fall 2005 UCF