CAP 5415 Computer Vision
Fall 2005

Dr. Alper Yilmaz
Univ. of Central Florida
www.cs.ucf.edu/courses/cap5415/fall2005
Office: CSB 250

Midterm 1

- 100<=x **3** **9.4%**
- 90<=x<100 **11** **34.71%**
- 80<=x<90 **10** **31.25%**
- 70<=x<80 **5** **15.62%**
- 60<=x<70 **3** **9.37%**

Recap
Circle Fitting

- Circle equation
  - Change x_0, and y_0 compute r

    $(x - x_o)^2 + (y - y_o)^2 - r^2 = 0$
  - Change only r and compute (x_0, y_0) from image gradient angle θ

    $x_0 = x - r\cos\theta \quad y_0 = y - r\sin\theta$
- Increment (x_0, y_0 , r) in accumulator array
- Find the local maxima

Alper Yilmaz, Fall 2005 UCF

Recap
Generalized Hough Transform

- For shapes with ***no*** analytical expression
- Requires learning of r-table
  - Distance vector and its angle for each boundary pixel to object centroid
- Finding a shape in an image
  - For input edge-map compute angle (can be from image gradient)
  - For each distance vector corresponding to edge angle increment a 2D accumulator array corresponding to (x_0, y_0)

Alper Yilmaz, Fall 2005 UCF

Recap
Medial Axis Transform

- Represents object shape: skeleton of an object
- Computed using an iterative algorithm,
- Inverse transform exist

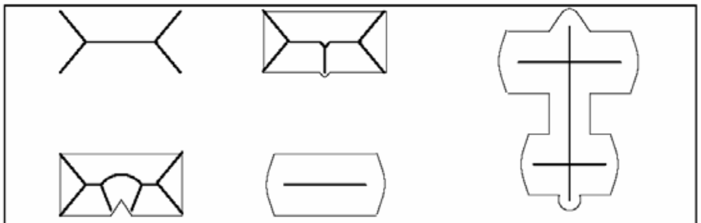


Figure 4.24: Some examples of medial axis transform.

Recap
Interest Points

- High texture variation in neighborhood of a pixel
- Movarec’s operator
  - Compute directional intensity variations in 4x4 neighborhood
  - Pick points with high intensity variations in a neighborhood
- Harris corner detector
  - Compute a moment matrix M from gradients in a neighborhood
  - Min eigenvalue of M higher than a threshold indicates corner

Object Motion

Alper Yilmaz, Fall 2005 UCF

Motion

- Projection of object motion in **real-world** (3D) results in motion in **image plane** (2D)
- 2D motion is defined over a sequence of frames
- Computed by using brightness constancy constraint
  - Intensity of a moving pixel does not change over-time

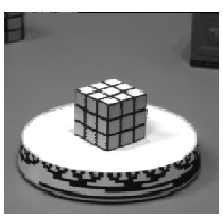
$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)$$

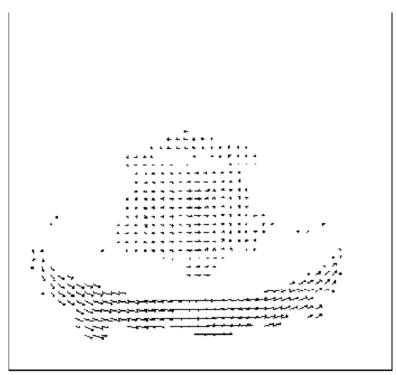
Alper Yilmaz, Fall 2005 UCF

What is its use?

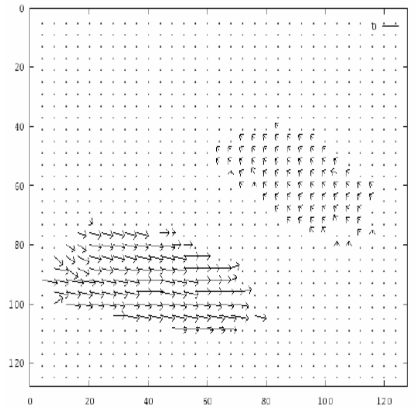
- Lots of uses
  - Motion Detection
  - Track objects
  - Correct for camera jitter (stabilization)
  - Align images (mosaics)
  - 3D shape reconstruction
  - Video Compression

Measurement of motion at every pixel





Measurement of motion at every pixel



Alper Yilmaz, Fall 2005 UCF

Visual Mosaics

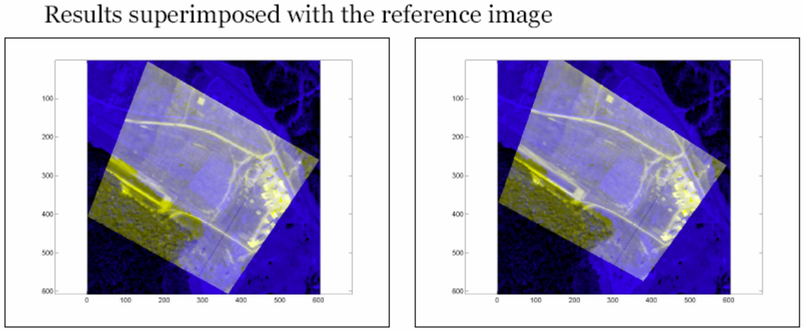
Alper Yilmaz, Fall 2005 UCF

Visual Mosaics

Alper Yilmaz, Fall 2005 UCF

Geo Registration

Results superimposed with the reference image

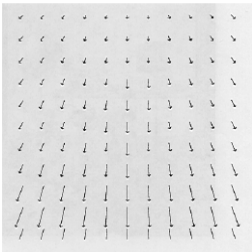


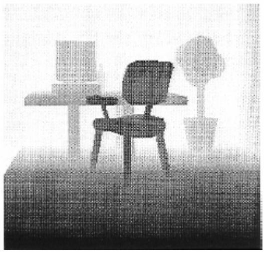
Alper Yilmaz, Fall 2005 UCF

Video Segmentation

Alper Yilmaz, Fall 2005 UCF

Structure From Motion

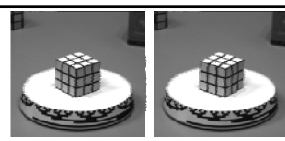




Alper Yilmaz, Fall 2005 UCF

Optical Flow

Alper Yilmaz, Fall 2005 UCF



Optical Flow

- Flow vector in image space (2D)
- Taylor series expansion of right side around Δt

$$I(x,y,t) = \boxed{I(x+\Delta x, y+\Delta y, t+\Delta t)}$$

$$\boxed{I(x+\Delta x, y+\Delta y, t+\Delta t)} = I(x,y,t) + \Delta x \frac{\partial I}{\partial x} + \Delta y \frac{\partial I}{\partial y} + \Delta t \frac{\partial I}{\partial t}$$

$$I(x,y,t) = I(x,y,t) + \Delta x I_x + \Delta y I_y + \Delta t I_t$$

$$0 = \Delta x I_x + \Delta y I_y + \Delta t I_t \Rightarrow 0 = \frac{\Delta x}{\Delta t} I_x + \frac{\Delta y}{\Delta t} I_y + I_t$$

Alper Yilmaz, Fall 2005 UCF

Optical Flow

$$0 = \frac{\Delta x}{\Delta t} I_x + \frac{\Delta y}{\Delta t} I_y + I_t \qquad u = \frac{\Delta x}{\Delta t} \qquad v = \frac{\Delta y}{\Delta t}$$

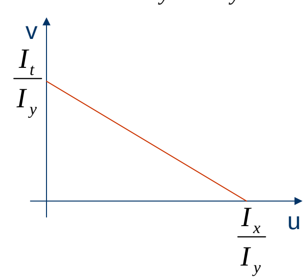
$$u I_x + v I_y + I_t = 0$$ Brightness constancy equation

$$v = u \frac{I_x}{I_y} + \frac{I_t}{I_y}$$ Equation of a line in (u,v) space

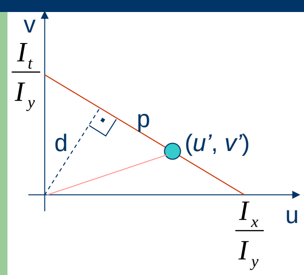
Optical Flow

- We know I_x, I_y, and I_t from images
- For every point these values provide one equation
- We have 2 unknowns: u, v
- Solution lie any where on the line

$$v = u \frac{I_x}{I_y} + \frac{I_t}{I_y}$$



Optical Flow



$$v = u\frac{I_x}{I_y} + \frac{I_t}{I_y}$$

- Let (u', v') be true flow
- True flow has two components
  - Normal flow: *d*
  - Parallel flow: *p*
- Normal flow **can be** computed
- Parallel flow **cannot**

Computing True Flow

- Horn & Schunck
- Schunk
- Lukas & Kanade

Horn & Schunck

$$uI_x + vI_y + I_t = 0$$

- Define an energy function and minimize

$$E(x,y) = (uI_x + vI_y + I_t)^2 + \lambda\overbrace{(u_x^2 + u_y^2 + v_x^2 + v_y^2)}^{f}$$

- Differentiate w.r.t. unknowns *u* and *v*

$$\frac{\partial E}{\partial u} = 2I_x(uI_x + vI_y + I_t) + \frac{\partial f}{\partial u} \qquad \frac{\partial f}{\partial u} = \frac{\partial}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial}{\partial u}\frac{\partial u}{\partial y} = 2(u_{xx} + u_{yy})$$

laplacian of *u*

$$\frac{\partial E}{\partial v} = 2I_y(uI_x + vI_y + I_t) + 2(v_{xx} + v_{yy})$$

laplacian of *v*

Horn & Schunck

$$I_x(uI_x + vI_y + I_t) + \Delta^2 u = 0 \qquad I_y(uI_x + vI_y + I_t) + \Delta^2 v = 0$$

- Laplacian controls smoothness of optical flow
  - A particular choice can be $\Delta^2 u = u - u_{avg}$, $\Delta^2 v = v - v_{avg}$.
- Rearranging equations

$$u(\lambda + I_x^2) + vI_xI_y + I_xI_t - \lambda u_{avg} = 0$$
$$v(\lambda + I_y^2) + uI_xI_y + I_yI_t - \lambda v_{avg} = 0$$

- 2 equations 2 unknowns
- Write v in terms of u
- Plug it in the other equation

Horn & Schunck Algorithm

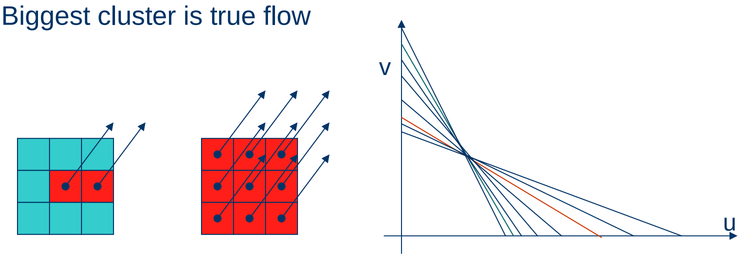
$$u = u_{avg} - I_x \left(\frac{I_x u_{avg} + I_y v_{avg} + I_t}{I_x^2 + I_y^2 + \lambda} \right) \qquad v = v_{avg} - I_y \left(\frac{I_x u_{avg} + I_y v_{avg} + I_t}{I_x^2 + I_y^2 + \lambda} \right)$$

- Iteratively compute u and v
  - Assume initially u and v are 0
  - Compute u_{avg} and v_{avg} in a neighborhood

Alper Yilmaz, Fall 2005 UCF

Schunck

- If two neighboring pixels move with same velocity
  - Corresponding flow equations intersect at a point in (u,v) space
  - Find the intersection point of lines
  - If more than 1 intersection points find clusters
  - Biggest cluster is true flow



Alper Yilmaz, Fall 2005 UCF

Lucas & Kanade

- Similar to line fitting we have seen
  - Define an energy functional
    - Take derivatives equate it to 0
    - Rearrange and construct an observation matrix

$$E = \sum (uI_x + vI_y + I_t)^2$$

$$\frac{\partial E}{\partial u} = \sum 2I_x(uI_x + vI_y + I_t) = 0$$

$$\frac{\partial E}{\partial v} = \sum 2I_y(uI_x + vI_y + I_t) = 0$$

Alper Yilmaz, Fall 2005 UCF

Lucas & Kanade 1

$$\frac{\partial E}{\partial u} = \sum 2I_x(uI_x + vI_y + I_t) = 0$$

$$\sum uI_x^2 + \sum vI_xI_y + \sum I_xI_t = 0$$

$$u\sum I_x^2 + v\sum I_xI_y = -\sum I_xI_t$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_xI_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\sum I_xI_t$$

$$\frac{\partial E}{\partial v} = \sum 2I_y(uI_x + vI_y + I_t) = 0$$

$$\sum uI_xI_y + \sum vI_y^2 + \sum I_yI_t = 0$$

$$u\sum I_xI_y + v\sum I_y^2 = -\sum I_yI_t$$

$$\begin{bmatrix} \sum I_xI_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\sum I_yI_t$$

$$\overbrace{\begin{bmatrix} \sum I_x^2 & \sum I_xI_y \\ \sum I_xI_y & \sum I_y^2 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} u \\ v \end{bmatrix}}^{u} = \overbrace{\begin{bmatrix} -\sum I_xI_t \\ -\sum I_yI_t \end{bmatrix}}^{B}$$

Alper Yilmaz, Fall 2005 UCF

Lucas & Kanade 2

$$Au = B \qquad A^{-1}Au = A^{-1}B \qquad Iu = A^{-1}B \qquad u = A^{-1}B$$

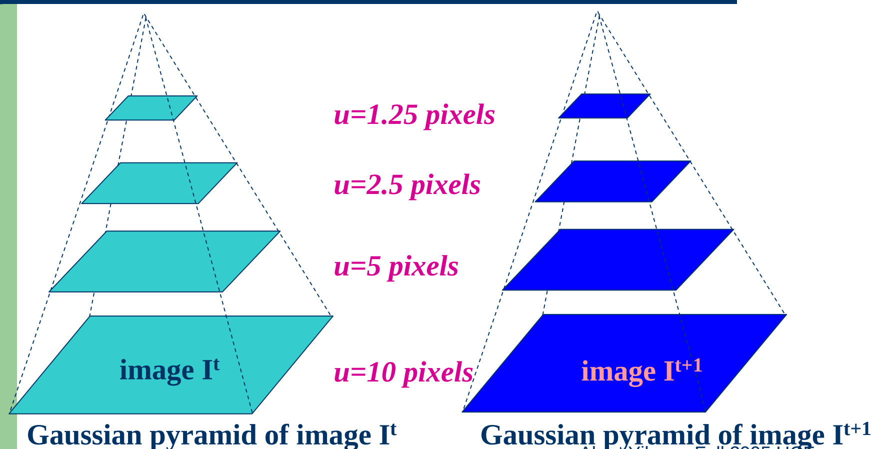
$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}^{-1} \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{\sum I_x^2 \sum I_y^2 - \left(\sum I_x I_y\right)^2} \begin{bmatrix} \sum I_y^2 & -\sum I_x I_y \\ -\sum I_x I_y & \sum I_x^2 \end{bmatrix} \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$$

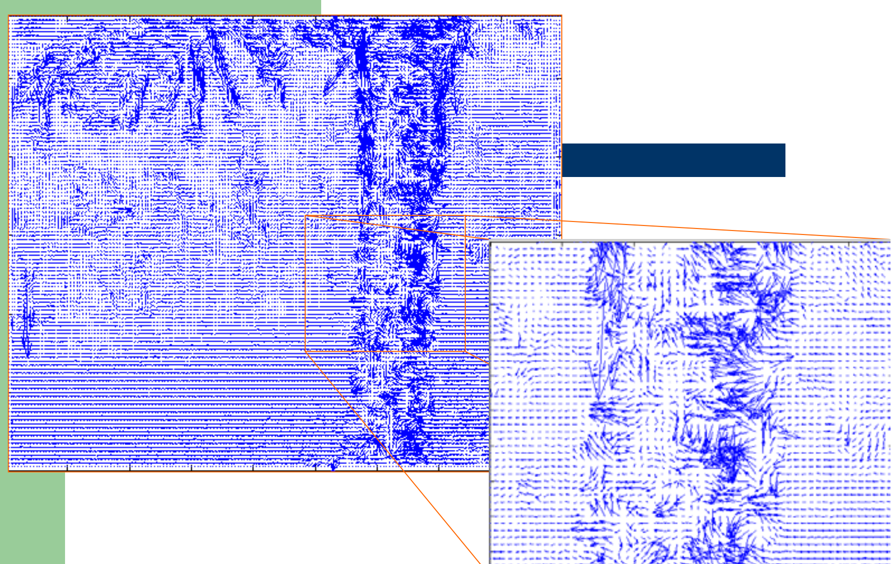
Discussion

- Horn-Schunck and Lucas-Kanade optical method works only for small motion.
- If object moves faster, the brightness changes rapidly, derivative masks fail to estimate spatiotemporal derivatives.
- Pyramids can be used to compute large optical flow vectors.

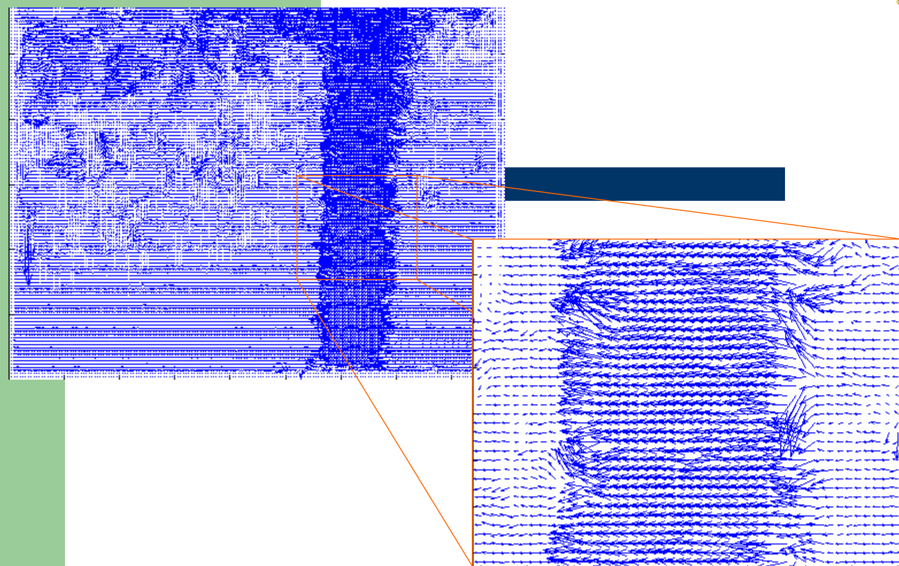
Coarse to Fine Optical Flow Estimation

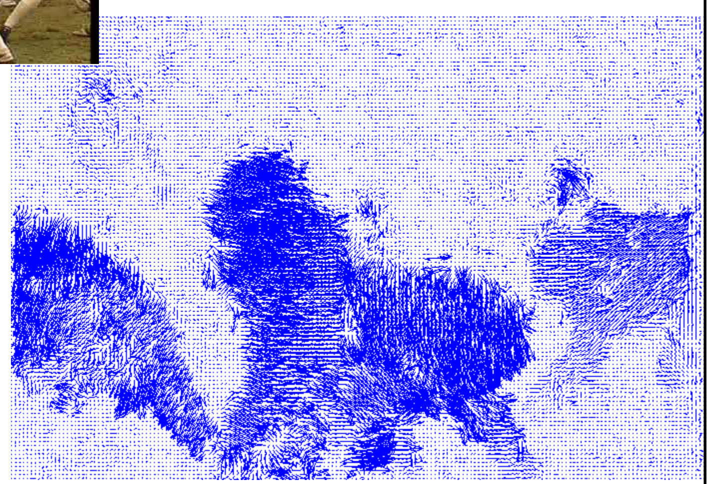


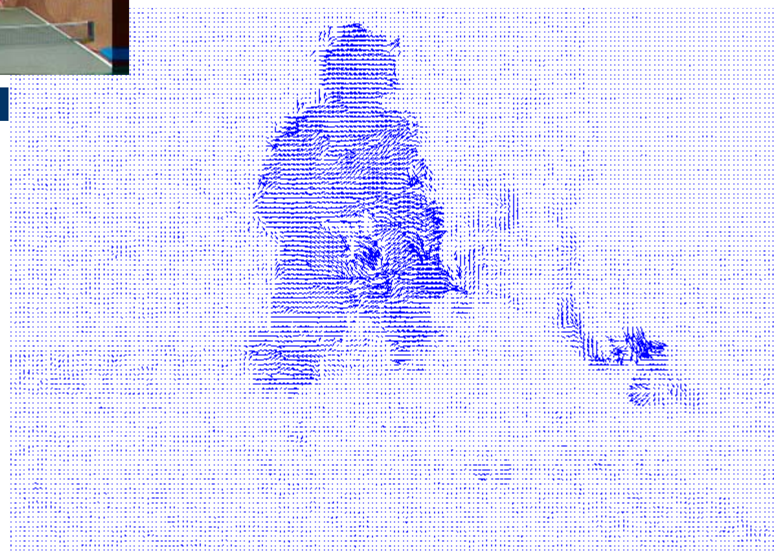
Alper Yilmaz, Fall 2005 UCF



Alper Yilmaz, Fall 2005 UCF







Alper Yilmaz, Fall 2005 UCF