CAP 5415 Computer Vision
Fall 2005

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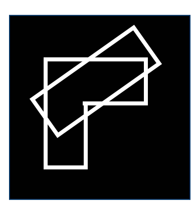
Recap
Line Fitting

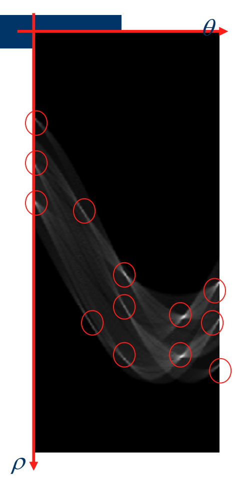
- For each edge point
  - Fix m compute

    $$b_i = y - m_j x$$

  - Fix θ compute

    $$x\cos\theta + y\sin\theta = \rho$$





Recap
Difficulties of Line Fitting

- What is the increments for θ and ρ.
- How many of detected lines are correct
- Which edge point belongs to which line
- Hardly ever satisfactory due to noise.

Recap
Line Fitting

- Alternatives
  - Least squares fit (take derivatives w.r.t. unknowns)
  - Define observation matrices, vector of unknowns

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & 1 \\ x_n & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{C} \qquad C = \left(A^T A\right)^{-1} A^T B$$

Circle Fitting

- Similar to line fitting
  - Three unknowns

$$(x - x_o)^2 + (y - y_o)^2 - r^2 = 0$$

- Construct a 3D accumulator array **A**
  - Dimensions: x_0, y_0, r
- Fix one of the parameters change the others
- Increment corresponding entry in **A**.
- Find the local maxima in **A**

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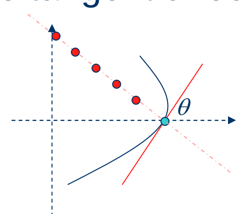
More Practical Circle Fitting

- Use the tangent direction θ at the edge point



- Compute x_0, y_0 given x, y, r

$$x_0 = x - r\cos\theta$$
$$y_0 = y - r\sin\theta$$

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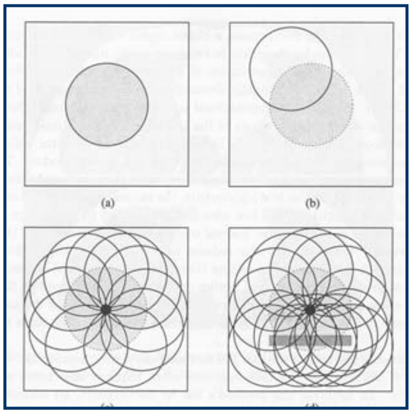
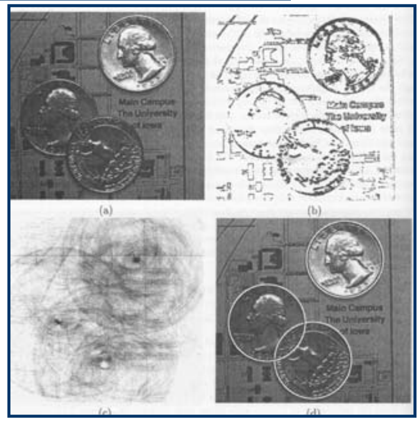
Examples





Generalized Hough Transform

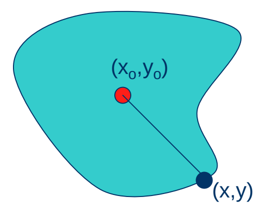
- Used for shapes with ***no*** analytical expression
- Requires training
  - Object of known shape
  - Generate model
    - R-table
- Similar approach to line and circle fitting during detection

Generating R-table

- Compute centroid
- For each edge compute its distance to centroid

$$r = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

- Find edge orientation (gradient angle)
- Construct a table of angles and *r* values



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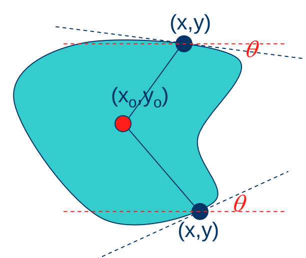
Generating R-table

θ1	r1, r2, r3 …
θ2	r14, r21, r23 …
θ3	r41, r42, r33 …
θ4	r10, r12, r13 …



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Detecting shape

- Knowns
  - Edge points (x,y)
  - Gradient angle at every edge point θ
  - R-table of shape need to be detected
- For each edge point find θ go to corresponding row of R-table
- Create an accumulator array of 2D (x,y)
  - Increment columns of θ

Rotation and Scale Invariance

- Rotation around Z-axis

$$x' = x\cos\alpha - y\sin\alpha$$
$$y' = x\sin\alpha + y\cos\alpha$$

- Scaling

$$x' = sx$$
$$y' = sy$$

- Rotation+scaling

$$x' = s(x\cos\alpha - y\sin\alpha)$$
$$y' = s(x\sin\alpha + y\cos\alpha)$$

Rotation and Scale Invariance

- Compute r are using rotation and scale

$$r = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s(x\cos\alpha - y\sin\alpha) - x_0 \\ s(x\sin\alpha + y\cos\alpha) - y_0 \end{bmatrix}$$

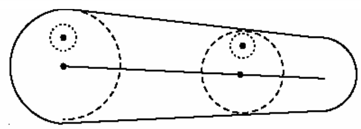
$$r = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

- Construct R-table for various values of α

Medial Axis Transform

Definition

- Shape representation
- Provides skeleton of an object
- Points on medial axis are the centers of maximal circular neighborhoods contained in the shape



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Examples

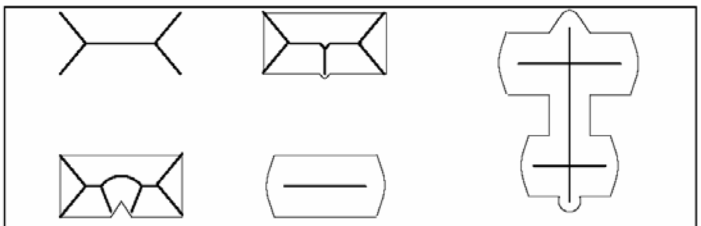

Figure 4.24: Some examples of medial axis transform.

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Iterative Algorithm

1. Iteratively compute f^k as follows:

$$f^k(x,y) = f^0(x,y) + \min(f^{k-1}(p,q))$$

$\forall(p,q)$ such that $distance((x,y),(p,q)) \leq 1$.

2. Medial axis is given by all points (x,y) such that:

$$f^k(x,y) \geq f^k(p,q),$$

$\forall(p,q)$ such that $distance((x,y),(p,q)) \leq 1$.

Figure 4.20: Iterative algorithm for computing medial axis transform.

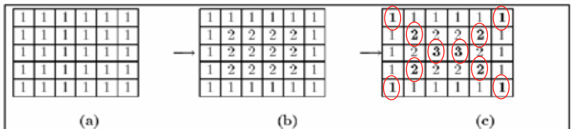

Figure 4.22: Medial axis transform. (a) Rectangular shape ($f^0(x,y)$). The background pixels, which are not shown, are all "0". (b) intermediate step ($f^1(x,y)$). (c) $f^2(x,y)$. The points in Medial axis shown in boldface.

Inverse Medial Axis Transform

1. Iteratively compute g^k as follows:

$$g^k(x,y) = \begin{cases} \max[0, (\max g^{k-1}(p,q)) - 1] & \text{if } g^{k-1} = 0 \\ g^{k-1} & \text{otherwise} \end{cases}$$

$\forall(p,q)$ such that $distance((x,y),(p,q)) \leq 1$.

Figure 4.21: Iterative algorithm for inverse medial axis transform.

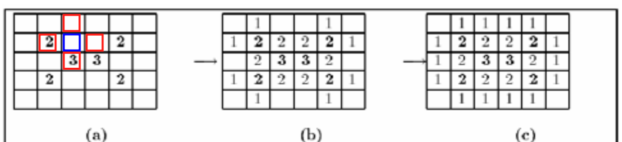

Figure 4.23: Inverse medial axis transform. (a) Medial axis ($g^0(x,y)$). (b) Intermediate step ($g^1(x,y)$). (c) Original shape ($g^2(x,y)$).

Interest Point Detectors

Movarec operator

Harris corner detector

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Interest Points

- Points that have important texture information in a local neighborhood.
  - T-joints, cross joints etc.

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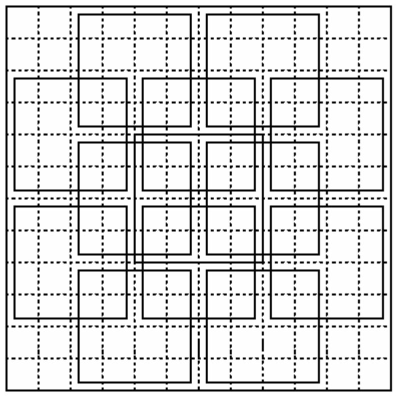
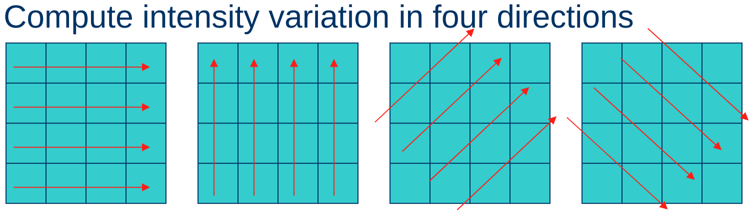
Movarec's Operator



Movarec's Operator

- Compute intensity variation in four directions



- Select the minimum of all direction to represent each 4x4 window
- If central window is higher or equal to all other windows mark point as interest point

Example

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Harris Corner Detector

- Compute the image derivatives I_x, I_y
- Construct gradient moment matrix

$$\Delta I = \begin{bmatrix} I_x \\ I_y \end{bmatrix} \quad M = \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix} \quad M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M_H = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

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Harris Corner Detector

$$M_H = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

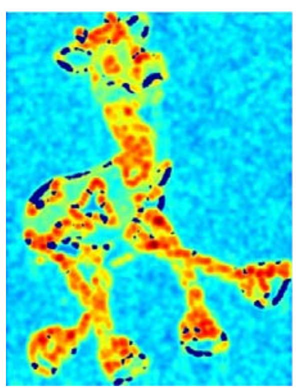
- Compute the eigenvalues of M_H
  - If both eigenvalues are high, this indicates a corner point

$$R = \min(eigenvalues(M_H)) > Th$$

- Same thing can be computed using

$$R = \det(M_h) + k\,\mathrm{trace}(M_h)^2 > Th$$

Example


Suggested Reading

- Chapter 4, Mubarak Shah, “Fundamentals of Computer Vision”