CAP 5415 Computer Vision
Fall 2005

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www.cs.ucf.edu/courses/cap5415/fall2005

Office: CSB 250

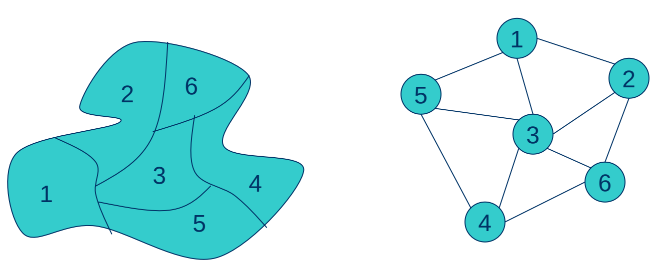
Recap
Steps in Seed Segmentation

1. Compute and smooth image histogram.
2. Detect good peaks and set thresholds
3. Apply connected component algorithm.
4. Merge small regions, split big regions

Recap
Region Growing Approaches

- Region splitting and merging
  - Region similarity function
- Phagocyte algorithm (boundary melting)
  - Compute intensity similarity between all the pixels of regions
- Likelihood ratio test
  - Two hypothesis: ∃ one region, ∃ two regions

Recap
Region Adjacency Graph



Geometric Properties of Regions

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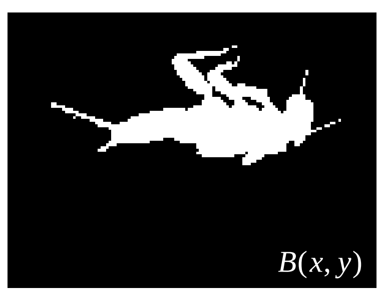
Region Properties

- Area
- Centroid
- Moments
- Perimeter
- Compactness
- Orientation



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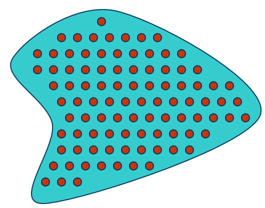
Area

- Number of pixels inside a region

$$A = \sum_{x=0}^{m} \sum_{y=0}^{n} B(x, y)$$

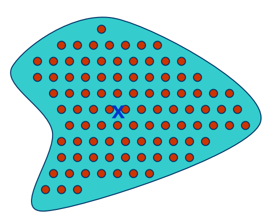


Centroid

- Center of mass

$$\bar{x} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} xB(x, y)}{A}$$

$$\bar{y} = \frac{\sum_{x=0}^{m} \sum_{y=0}^{n} yB(x, y)}{A}$$

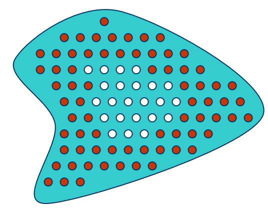


Moments

- Continuous

$$m_{pq}^{i} = \int \int x^p y^q B(x,y) dxdy$$

*i*th moment



- Discrete

$$m_{10}^{1} = \sum_{x=0}^{m} \sum_{y=0}^{n} xB(x,y), \; m_{01}^{1} = \sum_{x=0}^{m} \sum_{y=0}^{n} yB(x,y)$$

$$m_{20}^{2} = \sum_{x=0}^{m} \sum_{y=0}^{n} x^2 B(x,y), \;\; m_{02}^{2} = \sum_{x=0}^{m} \sum_{y=0}^{n} y^2 B(x,y), \;\; m_{11}^{2} = \sum_{x=0}^{m} \sum_{y=0}^{n} xyB(x,y)$$

Uniqueness Theorem

- The double moment sequence $\{m_{pq}\}$ is uniquely determined by $B(x,y)$ and conversely $B(x,y)$ is uniquely determined by $\{m_{pq}\}$

“Central” Moments

- Translation invariant
  - If same region appears in different places in the image its central moments are same

$$\mu_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q B(x,y)\ d(x-\bar{x})d(y-\bar{y})$$

Region centroid

Central phrase comes from centroid

Central Moments

$$\mu_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q B(x,y)\ d(x-\bar{x})d(y-\bar{y})$$

$$m^i_{pq} = \int \int x^p y^q B(x,y) dxdy$$

$$\mu_{00} = m_{00} \equiv \mu$$
$$\mu_{01} = 0$$
$$\mu_{10} = 0$$
$$\mu_{20} = m_{20} - \mu\bar{x}^2$$
$$\mu_{11} = m_{11} - \mu\bar{x}\bar{y}$$
$$\mu_{02} = m_{02} - \mu\bar{y}^2$$
$$\mu_{30} = m_{30} - 3m_{20}\bar{x} + 2\mu\bar{x}^3$$
$$\mu_{21} = m_{21} - m_{20}\bar{y} - 2m_{11}\bar{x} + 2\mu\bar{x}^2 y$$
$$\mu_{12} = m_{12} - m_{02}\bar{x} - 2m_{11}\bar{y} + 2\mu\bar{x}y^2$$
$$\mu_{03} = m_{03} - 3m_{02}\bar{y} + 2\mu\bar{y}^3$$

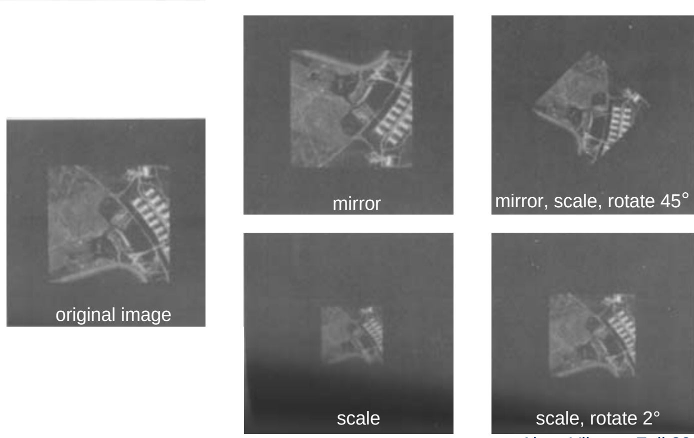
Hu Moments

$$\mu_{pq} = \int\int (x-\bar{x})^p (y-\bar{y})^q B(x,y)\ d(x-\bar{x})d(y-\bar{y})$$

- Translation, rotation, scaling invariant

$$\upsilon_1 = \mu_{20} + \mu_{02}$$
$$\upsilon_2 = (\mu_{20} - \mu_{02})^2 + {\mu_{11}}^2$$
$$\upsilon_3 = (\mu_{30} - 3\mu_{12})^2 + (3\mu_{12} - \mu_{03})^2$$
$$\upsilon_4 = (\mu_{30} + \mu_{12})^2 + (\mu_{21} + \mu_{03})^2$$
$$\vdots$$

Example

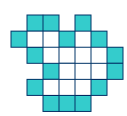


Example

Invariant (Log)	Original	Half Size	Mirrored	Rotated 2°	Rotated [illegible]
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

Perimeter and Compactness

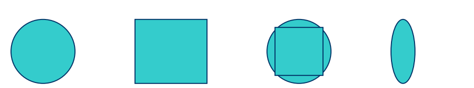
- Perimeter: Sum of pixels on the boundary of region



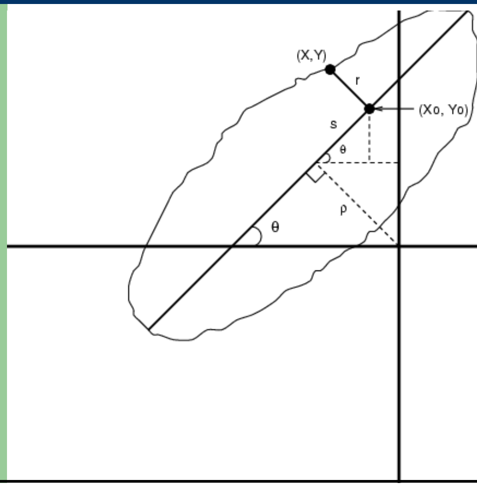
- Compactness
  - Circle is the most compact

$$C = \frac{P^2}{4\pi A}$$

perimeter (P), area (A)

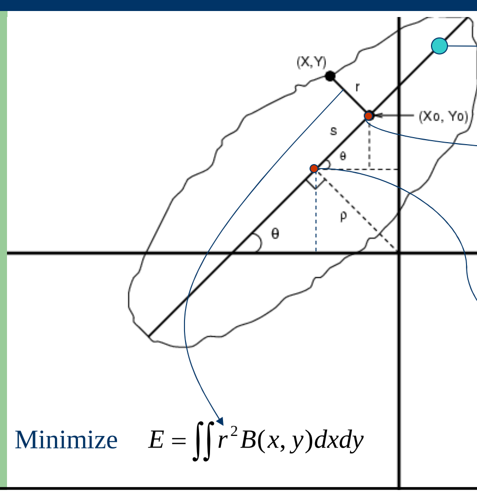


Region Orientation



Axis of 2nd moment

Region Orientation



line equation

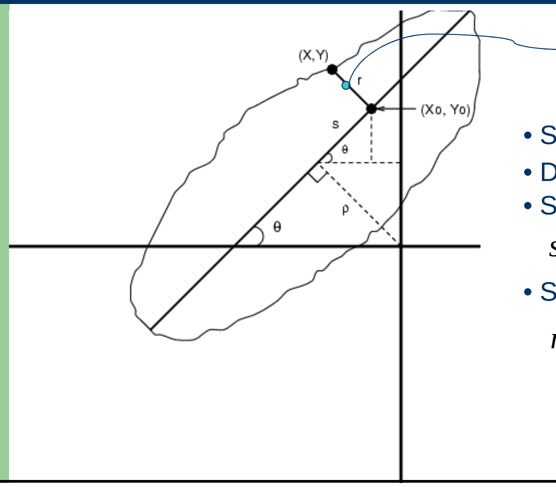
$$x\sin\theta - y\cos\theta + \rho = 0$$

$$x_0 = (-\rho\sin\theta) + s\cos\theta$$
$$y_0 = (\rho\cos\theta) + s\sin\theta$$

$$(-\rho\sin\theta, \rho\cos\theta)$$

Minimize $E = \iint r^2 B(x,y)\,dxdy$

Region Orientation



$$r^2 = (x - x_0)^2 + (y - y_0)^2$$

- Substituting (x_0, y_0) in r^2
- Differentiating w.r.t. s
- Setting it to 0

$$s = x\cos\theta + y\sin\theta$$

- Substituting s in (x_0, y_0)

$$r^2 = (x\sin\theta - y\cos\theta + \rho)^2$$

Region Orientation

$$r^2 = (x\sin\theta - y\cos\theta + \rho)^2$$

$$E = \iint r^2 B(x,y)dxdy$$

$$E = \iint (x\sin\theta - y\cos\theta + \rho)^2 B(x,y)dxdy$$

take derivative wrt ρ and equating it to 0→

$A(\bar{x}\sin\theta - \bar{y}\cos\theta + \rho) = 0$ where $(\bar{x}, \bar{y})$ is centroid

Using $x' = x - \bar{x},\ y' = y - \bar{y}$ we have $E = a\sin^2\theta - b\sin\theta\cos\theta + c\cos^2\theta$

or $E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2\theta - \frac{1}{2}b\sin 2\theta$

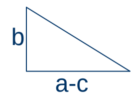
where $a = \iint x'^2 B(x,y)dx'dy',\ b = \iint x'y'B(x,y)dx'dy',\ c = \iint y'^2 B(x,y)dx'dy'$

Region Orientation

$$E = \frac{1}{2}(a+c) - \frac{1}{2}(a-c)\cos 2\theta - \frac{1}{2}b\sin 2\theta$$

take derivative wrt θ and equating it to 0→

$$\tan 2\theta = \frac{b}{a-c}$$



$$\sin 2\theta = \pm\frac{b}{\sqrt{b^2+(a-c)^2}}$$

$$\cos 2\theta = \pm\frac{a-c}{\sqrt{b^2+(a-c)^2}}$$

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Example

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Find: area, centroid, moments, compactness, perimeter, orientation

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MidTerm

- October 5

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