CAP 5415 Computer Vision
Fall 2005

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Recap
Estimation of Camera Parameters

- Relation between camera and image coordinates

$$x_i - o_x = -f_x \frac{r_{1,1}X_i + r_{1,2}Y_i + r_{1,3}Z_i + t_x}{r_{3,1}X_i + r_{3,2}Y_i + r_{3,3}Z_i + t_z} \qquad \textbf{(A)}$$

$$y_i - o_y = -f_y \frac{r_{2,1}X_i + r_{2,2}Y_i + r_{2,3}Z_i + t_y}{r_{3,1}X_i + r_{3,2}Y_i + r_{3,3}Z_i + t_z} \qquad \textbf{(B)}$$

- Estimate $r_{ij}, t_i, o_x, o_y, f_x, f_y$.

Recap
Estimation of Camera Parameters

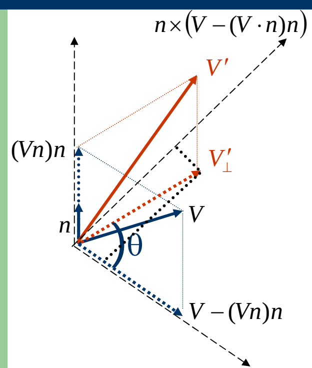
- Given corresponding world and image points
- Divide (A) to (B), rearrange result

$$x_i X_i v_1 + x_i Y_i v_2 + x_i Z_i v_3 + x_i v_4 - y_i X_i v_5 - y_i Y_i v_6 - y_i Z_i v_7 - y_i v_8 = 0 \quad \textbf{(C)}$$

$$v_1 = r_{21} \quad v_2 = r_{22} \quad v_3 = r_{23} \quad v_4 = t_y \quad v_5 = \alpha r_{11} \quad v_6 = \alpha r_{12} \quad v_7 = \alpha r_{13} \quad v_8 = \alpha t_x$$

- Rearrange into matrix and solve using SVD
- Estimate scale factor ➔ r_{2i} and t_y are there!!
- Compute α similar to scale factor
- Compute r_{3i} from r_{1i} and r_{2i}.
- Estimate f_x, f_y and t_z.
- Finally compute o_x and o_y from other knowns

Recap
Rotation around arbitrary axis



$$V = (V \cdot n)n + (V - (V \cdot n)n)$$

$$n \times V = n \times (V - (V.n)n)$$

$$n \times (n \times V) = (V.n)n - V$$

$$V'_\perp = \cos\theta (V - (V \cdot n)n) + \sin\theta (n \times V)$$

$$V' = V'_\perp + (V \cdot n)n$$

$$V' = -\cos\theta (n \times (n \times V)) + \sin\theta (n \times V) + n \times (n \times V) + V$$

Recap
Rotation around arbitrary axis

$$V' = V + \sin\theta(n \times V) + (1-\cos\theta)(n \times (n \times V))$$

$$n \times V = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \qquad n \times (n \times V) = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}\left(\begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}\right)$$

$$\begin{bmatrix} V_x' \\ V_y' \\ V_z' \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} + \cos\theta \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}\left(\begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}\right)$$

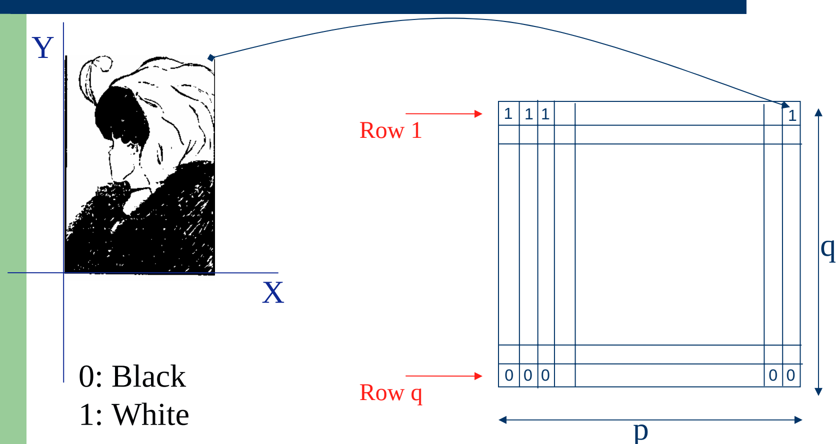
$$\begin{bmatrix} V_x' \\ V_y' \\ V_z' \end{bmatrix} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} + \sin\theta X(n)\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} + \cos\theta X^2(n)\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

Images

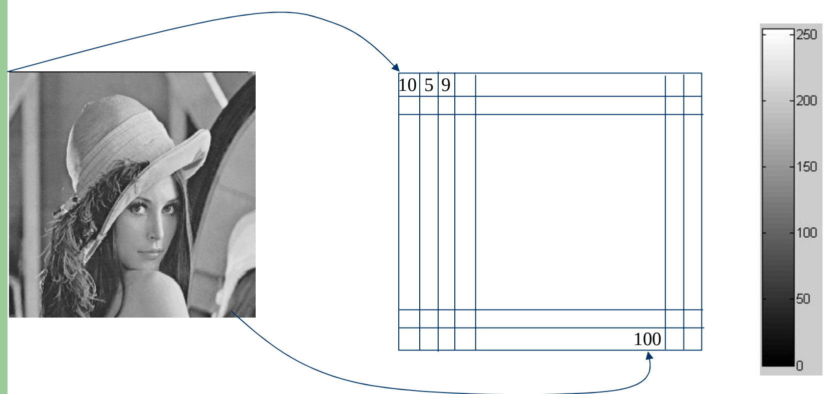
General

- Binary
- Gray Scale
- Color

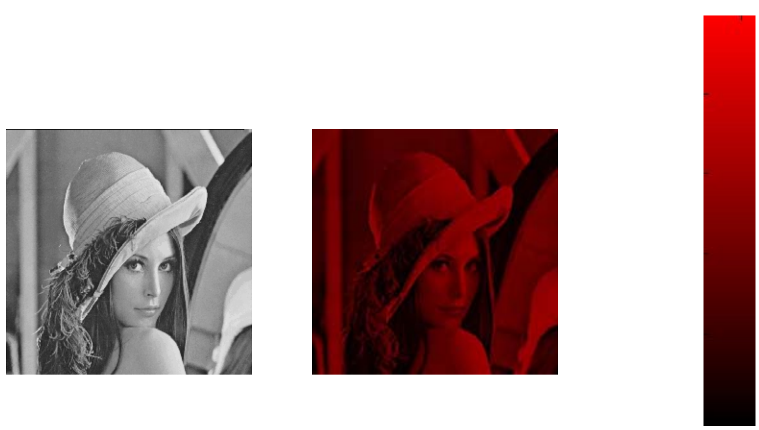
Binary Images



Gray Level Image



Gray Scale Image



Color Image
Red, Green, Blue Channels

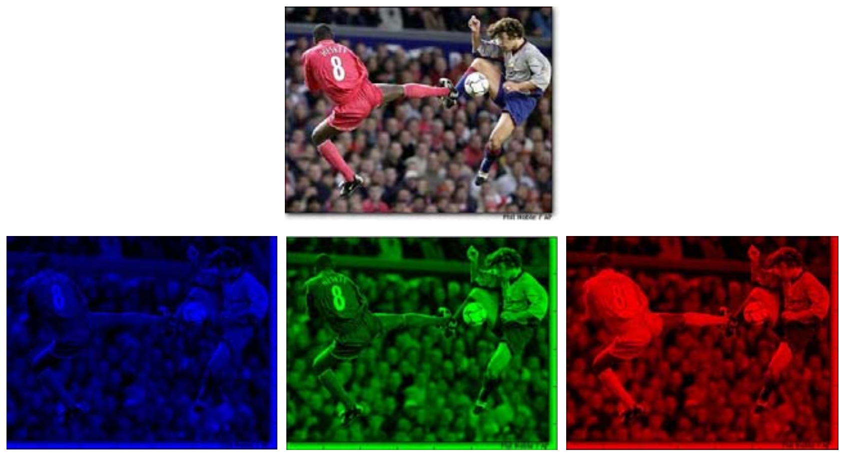


Image Histogram

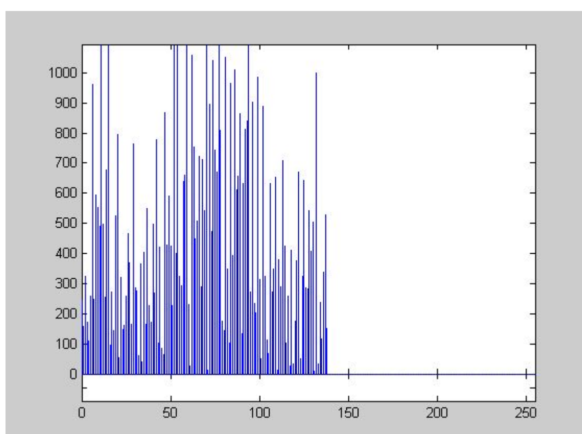


Image Noise

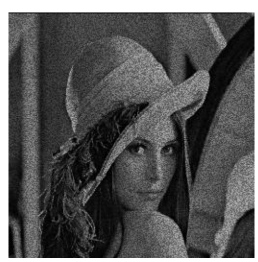
- Light Variations
- Camera Electronics
- Surface Reflectance
- Lens

Image Noise

- $I(x,y)$: the true pixel values
- $n(x,y)$: the noise at pixel (x,y)

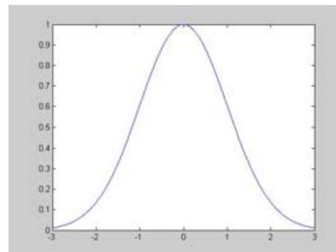
$$\hat{I}(x,y) = I(x,y) + n(x,y)$$

➔



Gaussian Noise

$$n(x,y) = e^{\frac{-n^2}{2\sigma^2}}$$



Salt & Pepper Noise

$$\hat{I}(x,y) = \begin{cases} I(x,y) & p < l \\ s_{\min} + r(s_{\max} - s_{\min}) & p \geq l \end{cases}$$

- p is uniformly distributed random variable
- l is threshold
- s_{min} and s_{min} are constant

Image Derivatives & Averages

Definitions

- Derivative: Rate of change
  - *Speed* is a rate of change of a *distance*
  - *Acceleration* is a rate of change of *speed*
- Average (Mean)
  - Dividing the sum of N values by N

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$v = \dfrac{ds}{dt}$ speed $\qquad a = \dfrac{dv}{dt}$ acceleration

Examples

$$y = x^2 + x^4 \qquad y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = 2x + 4x^3 \qquad \frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

Discrete Derivative Finite Difference

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$ Backward difference

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$ Forward difference

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$ Central difference

Example

$$
\begin{array}{lllllllll}
f(x) = & 10 & 15 & 10 & 10 & 25 & 20 & 20 & 20 \\
f'(x) = & 0 & 5 & -5 & 0 & 15 & -5 & 0 & 0 \\
f''(x) = & 0 & 5 & -10 & 5 & 15 & 20 & 5 & 0
\end{array}
$$

Derivative Masks

Backward difference	[-1 1]
Forward difference	[1 -1]
Central difference	[-1 0 1]

Derivatives in 2 Dimensions

Given function $f(x, y)$

Gradient vector $\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$

Gradient magnitude $|\nabla f(x, y)| = \sqrt{f_x^2 + f_y^2}$

Gradient direction $\theta = \tan^{-1} \frac{f_x}{f_y}$

Derivatives of Images

Derivative masks

$$f_x \Rightarrow \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad f_y \Rightarrow \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

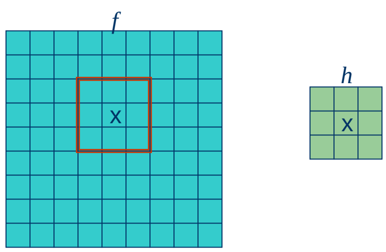
$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Derivatives of Images

$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Convolution

$$\begin{aligned} f*h = \; & f(x-1,y-1)h(-1,-1)+f(x,y-1)h(0,-1)+f(x+1,y-1)h(1,-1)+ \\ & f(x-1,y)h(-1,0)+f(x,y)h(0,0)+f(x+1,y)h(1,0) \\ & f(x-1,y+1)h(-1,1)+f(x,y+1)h(0,1)+f(x+1,y+1)h(1,1) \end{aligned}$$



$$f*h=\sum_{i=-1}^{1}\sum_{j=-1}^{1} f(x-i,y-i)h(i,j)$$

Averages

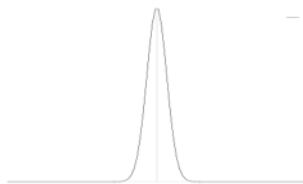
- Mean

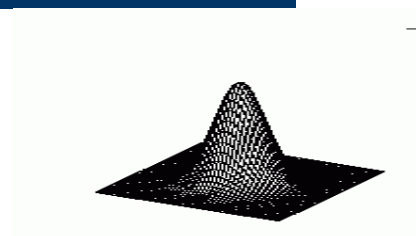
$$I=\frac{I_1+I_2+\ldots I_n}{n}=\frac{\sum_{i=1}^{n} I_i}{n}$$

- Weighted mean

$$I=\frac{w_1 I_1+w_2 I_2+\ldots+w_n I_n}{n}=\frac{\sum_{i=1}^{n} w_i I_i}{n}$$

Gaussian Filter





$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

$$g(x,y) = e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

Properties of Gaussian

- Most common natural model
- Smooth function, it has infinite number of derivatives
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- There are cells in eye that perform Gaussian filtering.