CAP 5415 Computer Vision
Fall 2005

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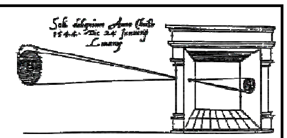


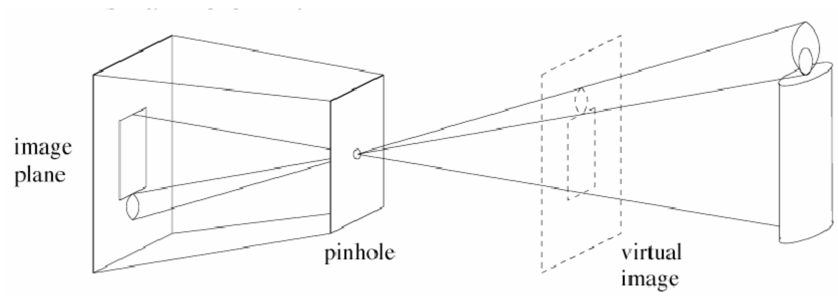
From "De radio astronomico et geometrico liber", 1545

Imaging Geometry

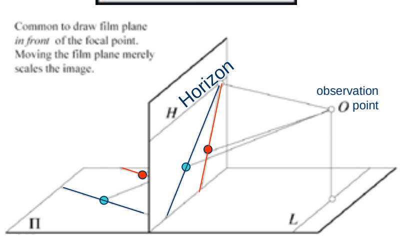
"Who would believe that so small a space could contain the image of all the universe? O mighty process! What talent can avail to penetrate a nature such as these? What tonque will it be that can unfold so great a wonder? Verily, none! This it is that guides the human discource to the considering of divine things. Here the figures, here the colors, here all the images of every part of the universe are contracted to a point. O what a point is so marvelous!"

Leonardo da Vinci (1485)

Pin hole Camera

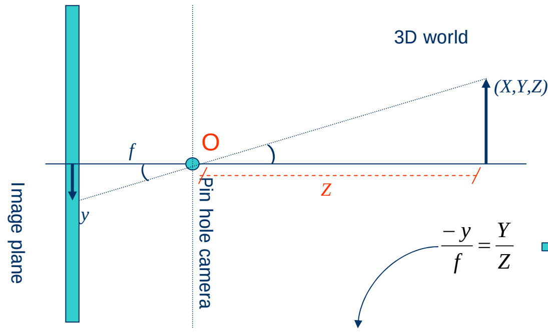


Perspective Projection



- Objects farther appear smaller
- Points go to Points
- Lines go to Lines
- Planes go to whole image or Half-planes
- Polygons go to Polygons

Perspective Projection (Origin at lens center)

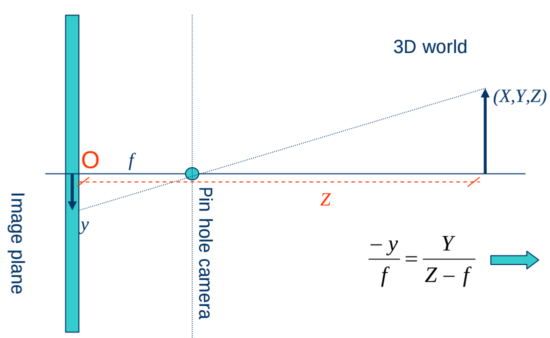


$$\frac{-y}{f} = \frac{Y}{Z} \Rightarrow y = -f\frac{Y}{Z}$$

Similar triangles

$$x = -f\frac{X}{Z}$$

Perspective Projection (Origin at image center)



$$\frac{-y}{f} = \frac{Y}{Z-f} \Rightarrow y = -f\frac{Y}{Z-f} \Rightarrow y = \frac{fY}{f-Z}$$

$$x = \frac{fX}{f-Z}$$

Homogenous Coordinates

$$\underbrace{(X,Y,Z)}_{\text{cartesian world coordinates}} \Rightarrow \underbrace{(kX,kY,kZ,k)}_{\text{homogenous world corrdinates}}$$

$$\underbrace{(C_1,C_2,C_3,C_4)}_{\text{homogenous world corrdinates}} \Rightarrow \underbrace{(\frac{C_1}{C_4},\frac{C_2}{C_4},\frac{C_3}{C_4})}_{\text{cartesian world coordinates}}$$

- k is any number.
- k=0 describes point at infinity (intersection of two parallel lines)

Camera Parameters

- Image coordinates (x_{image}, y_{image})
- Image center (o_x, o_y)
- Camera coordinates (x_{camera}, y_{camera})
- Real world coordinates (X, Y, Z)
- Focal length f
- Effective size of pixel in millimeter (k_x, k_y)

Camera Parameters

$$\boxed{\begin{aligned} x_{image} &= k_x x_{camera} + o_x \\ y_{image} &= k_y y_{camera} + o_y \end{aligned}}$$

scaling and translation

$$\begin{bmatrix} x_{image} \\ y_{image} \\ 1 \end{bmatrix} = \begin{bmatrix} k_x & 0 & o_x \\ 0 & k_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{camera} \\ y_{camera} \\ 1 \end{bmatrix}$$

due to perspective projection

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} k_x & 0 & o_x \\ 0 & k_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{camera} \\ V_{camera} \\ S \end{bmatrix} \qquad \boxed{-fX} \quad \boxed{-fY} \quad \boxed{Z}$$

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Camera Parameters

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} k_x & 0 & o_x \\ 0 & k_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{camera} \\ V_{camera} \\ S \end{bmatrix} \Rightarrow \begin{aligned} U_{camera} &= -f \frac{X_{camera}}{Z_{camera}} \\ V_{camera} &= -f \frac{Y_{camera}}{Z_{camera}} \end{aligned}$$

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} k_x & 0 & o_x \\ 0 & k_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix}$$

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Camera Parameters

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} k_x & 0 & o_x \\ 0 & k_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} -fk_x & 0 & o_x & 0 \\ 0 & -fk_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix}$$

$\boxed{f_x = fk_x}$

$\boxed{f_y = fk_y}$

Intrinsic Camera Parameters

- f_x
- f_y
- o_x
- o_y
- Intrinsic parameters do not depend on camera position in real world.

Extrinsic Camera Parameters

- Defined by orientation of camera in real world
  - Translation (3x1 vector)
  - Rotation (3x3 matrix)

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Translation

- (t_x, t_y, t_z) Translation vector

$$\begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \end{bmatrix} = \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \Rightarrow \begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Translation Matrix}} \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix} = T \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \\ 1 \end{bmatrix}$$

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Translation

- Inverse translation

$$T = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TT^{-1} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

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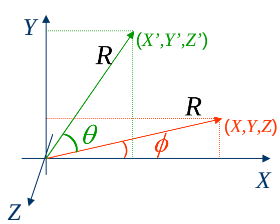
Rotation

- Around *Z*-axis



$$X = R\cos\phi$$

$$Y = R\sin\phi$$

$$X' = R\cos(\phi+\theta) = \overbrace{R\cos\phi}^{X}\cos\theta - \overbrace{R\sin\phi}^{Y}\sin\theta$$

$$Y' = R\sin(\phi+\theta) = \overbrace{R\cos\phi}^{X}\sin\theta + \overbrace{R\sin\phi}^{Y}\cos\theta$$

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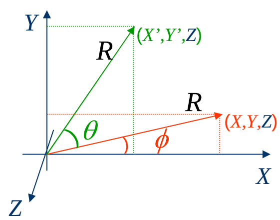
Rotation

- Around Z-axis



$$X' = X\cos\theta - Y\sin\theta$$

$$Y' = X\sin\theta + Y\cos\theta$$

$$\begin{bmatrix} X' \\ Y' \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

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Rotation

- Around X-axis

$$R^X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

- Around Y-axis

$$R^Y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

- Around Z-axis

$$R^Z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- No rotation

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Rotation

- Inverse rotation

$$R^Z.(R^Z)^T = I$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotation matrices are orthonormal!!

$$R_i^T \cdot R_j = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$$

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Euler Angles

- Let γ, β, α be rotation angles around X, Y, Z axis respectively.

$$R = R_Z^\alpha R_Y^\beta R_X^\gamma$$

- If angle θ is small, then $\cos\theta = 1$ and $\sin\theta = \theta$

$$R = \begin{bmatrix} 1 & -\alpha & -\beta \\ \alpha & 1 & -\gamma \\ \beta & \gamma & 1 \end{bmatrix}$$

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Camera Parameters (Revisited)

- World and camera coordinates
- t_x, t_y, t_z and $r_{1,1} \ldots r_{3,3}$ are extrinsic camera parameters

$$\mathbf{X}_{camera} = R\mathbf{X}_{world} + T$$

$$\begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & t_x \\ r_{2,1} & r_{2,2} & r_{2,3} & t_y \\ r_{3,1} & r_{3,2} & r_{3,3} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \\ 1 \end{bmatrix}$$

Camera Parameters (Revisited)

- World and image coordinates

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} -fk_x & 0 & 0 & o_x \\ 0 & -fk_y & 0 & o_y \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{camera} \\ Y_{camera} \\ Z_{camera} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} U_{image} \\ V_{image} \\ S \end{bmatrix} = \begin{bmatrix} -fk_x & 0 & 0 & o_x \\ 0 & -fk_y & 0 & o_y \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & t_x \\ r_{2,1} & r_{2,2} & r_{2,3} & t_y \\ r_{3,1} & r_{3,2} & r_{3,3} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{world} \\ Y_{world} \\ Z_{world} \\ 1 \end{bmatrix}$$

Camera Model

$$x_{image} - o_x = -f_x \frac{r_{1,1}X_{world} + r_{1,2}Y_{world} + r_{1,3}Z_{world} + t_x}{r_{3,1}X_{world} + r_{3,2}Y_{world} + r_{3,3}Z_{world} + t_z}$$

$$y_{image} - o_y = -f_y \frac{r_{2,1}X_{world} + r_{2,2}Y_{world} + r_{2,3}Z_{world} + T_y}{r_{3,1}X_{world} + r_{3,2}Y_{world} + r_{3,3}Z_{world} + T_z}$$

- Homework 1 (Due date: September 7, 2005)
  1. Derive “Euler angles” representation of rotation matrix (slide 20).
  2. Derive the “camera model” (slide 23).
- Reading
  - Text book (Trucco&Verri): Pages 26-40
  - Class notes: Pages 5-11

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