Solution 3.8.1.7

Applying the Laplace transform, assuming zero initial conditions, yields
\[ s^2 Y(s) + 11sY(s) + 10Y(s) = U(s), \]
or
\[ (s^2 + 11s + 10)Y(s) = U(s). \]
Rearranging yields
\[ \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 11s + 10}. \]

Solution 3.8.1.13

Applying the Laplace transform, assuming zero initial conditions, yields
\[ s^4 Y(s) + 15s^3Y(s) + 74s^2Y(s) + 135sY(s) + 72Y(s) = U(s) + sU(s), \]
or
\[ (s^4 + 15s^3 + 74s^2 + 135s + 72)Y(s) = U(s)(s + 1). \]
Rearranging yields
\[ \frac{Y(s)}{U(s)} = \frac{s + 1}{s^4 + 15s^3 + 74s^2 + 135s + 72}. \]
**Solution 3.8.3.2**

Let

\[
Z_1 = \frac{R_1}{C_1 s} \left( \frac{R_1}{R_1 + \frac{1}{C_1 s}} \right) = \frac{R_1}{R_1 C_1 s + 1}
\]

and

\[
Z_2 = R_2 + \frac{1}{C_2 s} = \frac{R_2 C_2 s + 1}{C_2 s}
\]

Then

\[
\frac{Z_2}{Z_1} = \frac{R_2 C_2 s + 1}{C_2 s} \times \frac{(C_2 s)(R_1 C_1 s + 1)}{(C_2 s)(R_1 C_1 s + 1)} = \frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{R_1 C_2 s}
\]

\[
= \frac{(R_2 / C_1)(s + (1 / R_1 C_1) s + (1 / R_2 C_2))}{s}
\]

We next have to choose some values. We know that

\[
\frac{R_2}{C_1} = 10
\]

\[
\frac{1}{R_1 C_1} = 0.1
\]

\[
\frac{1}{R_2 C_2} = 1.
\]

If we choose

\[
R_1 = 10^6 \Omega \quad C_1 = 10^{-4} \text{f} \quad R_2 = 10^5 \Omega \quad \text{and} \quad C_2 = 10^{-5} \text{f}
\]

Then

\[
G(s) = \frac{-10(s + 0.1)(s + 1)}{s}
\]

and we have achieved a PID compensator.
Let

\[ Z_1 = R_1 \| C_1 \]
\[ = \frac{R_1}{R_1/C_1 + 1} \]
\[ = \frac{R_1}{R_1 + 1/C_1} \]
\[ = \frac{1}{R_1C_1 + 1} \]

and

\[ Z_2 = R_2 \| C_2 \]
\[ = \frac{R_2}{R_2/C_2 + 1} \]
\[ = \frac{R_2}{R_2 + 1/C_2} \]
\[ = \frac{R_2}{R_2C_2 + 1} \]

Then

\[ Z_1 + Z_2 = \frac{R_1}{R_1C_1 + 1} + \frac{R_2}{R_2C_2 + 1} \]
\[ = \frac{(R_1R_2C_2 + R_1) + (R_1R_2C_1 + R_2)}{(R_1C_1 + 1)(R_2C_2 + 1)} \]
\[ = \frac{R_1R_2C_1 + R_2C_2 + (R_1 + R_2)}{(R_1C_1 + 1)(R_2C_2 + 1)} \]

Then

\[ \frac{Z_1 + Z_2}{Z_1} = \frac{R_1R_2(C_1 + C_2)s + (R_1 + R_2)}{(R_1C_1 + 1)(R_2C_2 + 1)} \]
\[ = \frac{R_1R_2(C_1 + C_2)s + (R_1 + R_2)}{R_1(R_2C_2 + 1)} \]
\[ = \left( \frac{(C_1 + C_2)}{C_2} \right) s + \frac{R_1 + R_2}{R_1R_2(C_1 + C_2)} \]
\[ = \frac{1}{s + \frac{R_1R_2(C_1 + C_2)}{R_2C_2}} \]

We next have to choose some values. We note that we have

\[ \frac{C_1 + C_2}{C_1} = 10 \]
\[
\frac{R_1 + R_2}{R_1R_2(C_1 + C_2)} = 0.1
\]

\[
\frac{1}{R_2C_2} = 0.01
\]

From the first equation we see that

\[C_1 = 9C_2.\]

Our choices are fairly limited. If we choose

\[C_1 = 100 \mu \text{f} \quad \text{and} \quad C_2 = 10 \mu \text{f}\]

we get a gain of 11, which is close to what we want.

Then if we choose

\[R_2 = 10 \text{ M} \Omega,\]

the pole will be at

\[-\frac{1}{10^7 \times 10^{-5}} = -0.01,\]

We can easily make a 10 M \Omega resistor from the values in Table 3.1, and modern JFET opamps can tolerate a resistor this big, since the offsets have become very small, particularly if we are willing to pay about a buck for a quad opamp.

If we now solve the second equation for \(R_1\) we obtain

\[
R_1 = \frac{R_2}{0.1R_2(C_1 + C_2) - 1}
\]

\[
= \frac{10^7}{0.1 \times 10^7 \times 1.1 \times 10^{-4} - 1}
\]

\[
= \frac{10^7}{109}
\]

\[
= 91.7 \text{ k} \Omega \to 91 \text{ k} \Omega.
\]

Then

\[G(s) = \frac{11(s + 0.1008)}{s + 0.01}\]

Given that we normally use 5% components, this compensator is very close to the desired one. The gain is 10% high but easily adjusted at the power amplifier that will of needs be present.
Solution 3.8.5.4

For the opamp circuit of Figure 3.27 we have

\[
\frac{V_o(s)}{V_i(s)} = \frac{-(C_1/C)[s^2 + [(1/C_1)(1/R_1 - R_2/R_5)]s + 1/RR_4C_1C}{s^2 + (1/R_2C)s + 1/R^2C^2}
\]

We begin with the denominator which should be

\[(s + 0.1)(s + 50) = s^2 + 50.1s + 5.\]

Note that we have two nonlinear equations in three unknowns, namely

\[
\begin{align*}
\frac{1}{R_2C} &= 50.1 \\
\frac{1}{R^2C^2} &= 5.
\end{align*}
\]

Squaring the first equation and dividing by the second we obtain:

\[
\left(\frac{R}{R_2}\right)^2 = \frac{50.1^2}{5} = 502.002,
\]

or

\[
\frac{R}{R_2} = 22.4.
\]

If we choose

\[R_2 = 12 \text{ k } \Omega,\]

then

\[R = 22.4054 \times 12 \text{ k } \Omega = 268.9 \text{ k } \Omega.\]

From the table we choose

\[R = 270 \text{ k } \Omega.\]

We now have to have

\[
\frac{1}{R_2C} = 50.1,
\]

or

\[
C = \frac{1}{50.1 \times R_2} = \frac{1}{50.1 \times 12 \times 10^3} = 1.66 \mu \text{ f}.
\]
We can come close to this value with the parallel combination of three 0.22 μf capacitors and a 1.0 μf capacitor. We next check the other coefficient of the denominator to see if it is correct.

\[
\frac{1}{R^2C^2} = \frac{1}{270^2 \times 10^{10} \times 1.666^2 \times 10^{-12}} = 4.958.
\]

Thus the denominator polynomial is

\[s^2 + 50.1s + 4.958 = (s + 50)(s + 0.0992).\]

and so the roots are very close to those we want.

We now turn our attention to the numerator and the gain. The gain is

\[K = 10 = \frac{C_1}{C}.
\]

We can come very close with a parallel combination of a 10, 4.7, and two 1 μf capacitors. Thus, assuming that

\[C_1 = 16.7 \mu f,
\]

we next consider the coefficients of the numerator of the transfer function which is

\[(s + 1)(s + 5) = s^2 + 6s + 5.
\]

We thus have the two equations

\[
\begin{align*}
\frac{1}{R_1 - R_3/RR_5} &= 6C_1 \\
\frac{1}{RR_4C_1} &= 5
\end{align*}
\]

From the second equation we see that

\[R_4 = \frac{1}{5RC_1} = \frac{1}{5 \times 270 \times 10^3 \times 1.66 \times 10^{-6} \times 16.7 \times 10^{-6}} = 26.67 \text{ kΩ}.
\]

This is quite close to the value 27 kΩ from the table. Then

\[\frac{1}{RR_4C_1} = 4.9383
\]

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We now turn our attention to the last coefficient which is determined from the equation,

\[
\frac{1}{R_1} - \frac{R_3}{RR_5} = 6C_1,
\]
or

\[
\frac{1}{R_1} - \frac{R_3}{RR_5} = 0.0001002.
\]

If we choose

\[ R_1 = 10 \text{ k } \Omega, \]

then all we have to do is make

\[ \frac{R_3}{RR_5} \text{ very small.} \]

Suppose we choose

\[ R_3 = 10 \text{ k } \Omega, \]

and

\[ R_5 = 180 \text{ k } \Omega, \]

Then

\[
\frac{R_3}{RR_5} = \frac{10^4}{1.8 \times 10^5 \times 2.7 \times 10^5} = 2.05 \times 10^{-7}.
\]

Then the numerator polynomial is

\[ s^2 + 4.99s + 4.9383 = (s + 0.9906)(s + 4.99). \]

Close enough, and we have been able to set all the coefficients of the numerator and denominator, as well as the gain.
Solution 3.8.7.1

In part (a) the output is

\[ C = AG + B \]

In the block diagram of part (b) we have

\[ C = (A + BX)G \]
\[ = AG + BXG. \]

This says that

\[ X = G^{-1}. \]

The way to think about is to note that in part (a) \( B \) does not go through \( G \). In part (b) it does so we have condition \( B \) by multiplying by the inverse of \( G \).
**Solution 3.8.7.2**

In part (a) the output is

\[ C = A + AG \]

In the block diagram of part (b) we have

\[ C = AGX \]

Equating the two expressions for \( C \) yields

\[ AGX = A + AG, \]

which, when solved for \( X \) gives

\[ X = 1 + G^{-1}. \]

---

**Solution 3.8.7.3**

In part (a) the output is

\[ D = A + B - C \]

In the block diagram of part (b) we have

\[ D = (A + B) - X \]

Thus,

\[ X = C. \]

The point of this exercise is simply to show that we can separate a single summer into multiple summers and still have the same output.
In part (a) the output is

\[ D = (A - B) + C \]
\[ = A - B + C \]

In the block diagram of part (b) we have

\[ D = (A + X) - Y. \]

There are lots of answers but the simplest one is to let

\[ X = C \quad \text{and} \quad Y = B. \]

In other words, we can first add \( C \) to \( A \) and then subtract \( B \) and get the same result as first subtracting \( B \) from \( A \) and then adding \( C \).