1) Which of the following logical expressions is equivalent to $\neg p \land (p \rightarrow q)$
   a) $p$  b) $\neg p$  c) $q$  d) $\neg q$  e) None of the above

2) In class we showed how to represent a subset of a set using a single integer. For example, if our set is \{‘a’, ‘b’, ‘c’, ‘d’, ‘e’, ‘f’, ‘g’, ‘h’\}, the integer 10 can represent the set of ‘e’ and ‘g’, since, in binary, 10 is 00001010. Which of the following bitwise operations represents the intersection of the sets \{‘a’, ‘b’, ‘d’, ‘h’\} and \{‘b’, ‘c’, ‘e’, ‘g’\}?
   a) 209 | 106  b) 209 ^ 106  c) 204 & 110  d) 204 && 110  e) None of the Above

3) In class we looked at a Boolean formula that was true iff the corresponding Sudoku puzzle was valid. How many different Boolean variables were in the formula?
   a) 9  b) 81  c) 729  d) 6561  e) None of the Above

4) Which of the following represents dimensions of matrices that can be multiplied? (The order listed is the same order as the attempted multiplication, and a \(n \times m\) matrix has \(n\) rows and \(m\) columns.)
   a) 2 x 3 times 3 x 7  b) 2 x 4 times 2 x 4  c) 3 x 5 times 7 x 5  d) 4 x 6 times 8 x 3  e) None of the Above

5) If \(f(x) = (2x + 3)^2\) and \(g(x) = \sqrt{x}\), what is \(g(f(x))\)?
   a) \(2x + 3\)  b) \(|2x + 3|\)  c) \(4x + 6\sqrt{x} + 9\)  d) \(4x + 12\sqrt{x} + 9\)  e) None of the Above
Recitation #5: Free Response Problems

1/31/2014

1) Give a statement P(x, y) such that ∀x∃yP(x, y) is true but ∃x∀yP(x, y) is false. Make sure your statement is a mathematical one that can be proven completely using the techniques we’ve learned. (So, don’t come up with a regular English example…)

2) Use the Laws of Implication and Logic Laws to verify the following argument:

\[
(p \lor q) \rightarrow r \\
r \rightarrow (s \lor t) \\
s \land \bar{u} \\
\bar{u} \rightarrow \bar{t} \\
\therefore p
\]

3) Let A, B and C be sets such that \( A \subseteq B \). Prove that \((C - B) \subseteq (C - A)\).

4) For what domain and range is the function \( f(x) = \frac{x+1}{x+2} \) a bijection? In particular, choose each set as the set of Reals except one specific value, though that specific value will be different for the domain and range. Given these restrictions on the domain and range, prove that \( f(x) \) is a bijection.

5) Given the following Markov Chain, give a matrix expression that would help determine the probability of starting in state 1 and ending in state 3 after 17 steps: