Counting

Adding vs Multiplying

Total # people in room =
# women + # men

Anytime \( A \cap B = \emptyset \)
\[ |A \cup B| = |A| + |B|. \]

Can wrong in 2 ways
(a) miss some item
(b) count an item more than once.

\[ |A \times B| = (|A| \times |B|) \]

<table>
<thead>
<tr>
<th>entree</th>
<th>sides</th>
<th>combo</th>
<th>beer</th>
</tr>
</thead>
<tbody>
<tr>
<td>taco</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>burrito</td>
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<td></td>
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<tr>
<td>one pizza</td>
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<tr>
<td>gordita</td>
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<tr>
<td>crunchwrap</td>
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</table>
Subtracting Rule

Sometimes it's easier to subtract out of the whole, the items you don't want to count!

\[
2^{10} - 10 \cdot 2^9 - 10 \\
= 2^{10} - 1 - 10 \\
= 1024 - 11 \\
= 1013
\]

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]

Perm n items = \( n! \)

Combin combo \( = \frac{n!}{a_1!a_2!...a_k!} \)

Combinos with repetition

Buying n items and choosing k from k different types of items (with no restrictions) we can do this in

\[
\binom{n+k-1}{n} = \binom{n+k-1}{k-1}
\]
If I try to pick 129 things from 128 groups, I am forced to pick at least 2 items from some group.

Pigeonhole Principle
\( |A \cup B| = |A| + |B| - |A \cap B| \)

\( x_1 + x_2 + x_3 + x_4 + x_5 = 15 \)
\[ 0 \leq x_i \leq 15 \]

The difference between sets to this equation:

\[ 5 + 0 + 2 + 4 + 4 = 15 \]
\[ \text{bad} \quad 6! \quad 3 \quad \text{fech} \quad 5! \quad 3 \]

How many times do both \text{bad} and \text{fech} appear?

\[ \text{bad}, \text{fech}, \text{good} \quad 3! \]

\# times \text{bad OR fech appears} = 6! + 5! - 3!

\# orders w/o either = 8! - (6! + 5! - 3!)

= 8! - 6! - 5! + 3!