Suggested Set Questions from Old Foundation Exams

**Fall 2012** (15 pts) PRF (Sets)

Let A, B and C be finite sets of integers. Prove or disprove the following assertions:

a) If $C \subseteq A \cap B$, then $A - B \subseteq A - C$.

b) If $A \cap B \subseteq C$, then $(A - C) \cup (B - C) \cup (A \cap B) = A \cup B$.

**Fall 2013** (10 pts) PRF (Sets)

Let A, B and C be finite sets of integers. If $A \cap B = A \cap C$ and $\bar{A} \cap B = \bar{A} \cap C$, prove that $B = C$.

**Spring 2014** (15 pts) PRF (Sets)

Prove or disprove the following assertion about finite sets A and B:

$$P(A) \cap P(B) = P(A \cap B)$$

Recall that $P(A)$ is simply the set of all subsets of A.

**Fall 2014** (15 pts) PRF (Sets)

Prove the following statement is true:

Let A, B and C be three finite sets. Prove that $(A - B) - C \subseteq A - (B - C)$.

(Note: $-$ denotes differences of two sets.)
Let $R$ be a relation over the positive integers greater than 1 defined as follows: $R = \{ (x, y) \mid \gcd(x, y) > 1 \}$, namely, $x$ and $y$ share a common factor. Determine, with proof, whether or not $R$ satisfies the following properties: (i) reflexive, (ii) irreflexive, (iii) symmetric, (iv) anti-symmetric, (v) transitive.

Consider the following relation $R$, between UCF students:

$$R = \{ (x, y) \mid \text{there exists some instructor } z \text{ such that both } x \text{ and } y \text{ have taken classes from } z \}$$

We define a UCF student to be anyone who has taken at least one class from an instructor at UCF.

With proof, determine which of the following properties $R$ satisfies:

(a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric, and (e) transitive.

Let $R, S$ and $T$ be relations and $A, B$ and $C$ be finite sets with $R \subseteq A \times B$, $S \subseteq A \times B$, and $T \subseteq B \times C$. Prove that $T \circ R \cup T \circ S = T \circ (R \cup S)$. (Note, we define the compositions of two relations $S \circ R$ to mean applying $R$, followed by applying $S$.)

(a) (5 pts) Let $A = \{1, 2, 3, 4\}$, $B = \{x, y, z\}$ and $C = \{m, n\}$. Let $R = \{(1, x), (1, z), (2, y), (4, x), (4, y), (4, z)\}$ and $S = \{(x, m), (y, m), (y, n)\}$. What is $S \circ R$?

(b) (5 pts) Let $A = \{1, 2, 3\}$. There are 5 equivalence relations over $A \times A$. Explicitly list all five.