1) Prove that the set of ordered pairs (x, y), where x and y are positive integers are countable.

2) Prove that the subsets of the positive integers are uncountable.

3) Prove $\text{EQ}_{\text{DFA}}$ is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works and show why this is good enough.

4) Find a match in the following instance of the PCP: $\left\{ \left[ \begin{array}{c} ab \\ a b a b \end{array} \right], \left[ \begin{array}{c} b \\ a \end{array} \right], \left[ \begin{array}{c} a b a \\ b \end{array} \right], \left[ \begin{array}{c} a a \\ a \end{array} \right] \right\}$.

5) Let $S = \{ <M> \mid M$ is a TM that accepts $w^R$ whenever it accepts $w \}$. Show that $S$ is undecidable.

6) Consider the problem of testing whether a Turning machine $M$ on an input $w$ ever attempts to move its head left at any point during its computation on $w$. Formulate this problem as a language and show that it’s decidable.