Explanation of transitions:

$q_0 \rightarrow q_0$: In all 3 of these, the first two bits add to the third, maintaining equality.

$q_0 \rightarrow q_1$: The parity of the bottom bit is the same as the sum of the top two, but a carry bit is produced, so the current string is no longer valid.

$q_0 \rightarrow q_2$: The parity of the answer bit is incorrect. All strings with this prefix are NOT in the designated language.

$q_1 \rightarrow q_0$: This fixes the previous parity carry bit problem.

$q_1 \rightarrow q_1$: The carry bit has cascaded to the next bit.

$q_1 \rightarrow q_2$: The current bit is now incorrect, so all strings with this prefix are NOT in the language.

8) $L_1: 1(011)^*0$

$L_2: (011)^*0101(011)^*$


$L_4: EUI(011)(011)(011)$

$L_5: (0^{10*})(0^{10*}10^{*})^*U10^{*}10^{*}$

Note: For each of these, it's important to include all possible strings that fit the given criteria.
9) \[ D_1: \]
![Diagram of D_1]

9a) \[ \text{GNFA-4:} \]
![Diagram of GNFA-4]

9b) \[ \text{RIP out 1:} \]
![Diagram of RIP out 1]

9c) \[ \text{RIP out 2:} \]
![Diagram of RIP out 2]

Thus \( a \cdot R \cdot E \cdot \) that expresses the same language as \( D_1 \) is

\[ a^*b(a \cup ba^*b)^* \]

\[ D_2: \]
![Diagram of D_2]

\[ \text{GNFA-5:} \]
![Diagram of GNFA-5]

\[ \text{RIP out 1:} \]
![Diagram of RIP out 1]

\[ \text{RIP out 2:} \]
![Diagram of RIP out 2]

\[ \text{RIP out 3:} \]
![Diagram of RIP out 3]

Final B.E.: \[ \varepsilon \cup (a \cup b)a^*b[(b \cup a(a \cup b))a^*b]^*(a \cup \varepsilon) \]
1) We will show that $L_1$ is not regular by showing that it does not satisfy the pumping lemma. Let $p$ be the pumping length for $L_1$, assuming it's regular. Consider $S = 0^p 1^p 2^p$. The pumping lemma claims that $S = xyz$, with $|xy| \leq p$, $|y| > 0$. Then $x = 0^i$, $y = 0^j$, $z = 0^{p-i-j} 1^p 2^p$.

Consider $xy^2z = 0^{p+j} 1^p 2^p \not\in L_1$, since $j > 0$. This contradicts the pumping lemma. Thus $L_1$ is not regular.

We will show that $L_2$ is not regular by showing that it does not satisfy the pumping lemma. Let $p$ be the pumping length for $L_2$, assuming it's regular. Consider $S = 0^p 1^p 10^p$. The pumping lemma claims that $S = xyz$, with $|xy| \leq p$, $|y| > 0$. Then $x = 0^i$, $y = 0^j$, $z = 0^{p-i-j} 1^p 10^p$.

Consider $xy^3z = 0^{p+3j} 10^p$. The length of this string is $2(p+j+1)$. Since $p+j+1 \leq p+3j+p+2j$, it follows that the first half of this string is all Os while the second half is not. Thus, $xy^3z \not\in L_2$, contradicting the pumping lemma, proving $L_2$ is not regular.

We will show that $L_3$ is not regular by showing that it does not satisfy the pumping lemma. Let $p$ be the pumping length and consider the string $S = 0^p$. We must have $S = xyz$, with $x = 0^i$, $y = 0^j$, $z = 0^{p-i-j}$ and $1 \leq j \leq p$. Consider $xy^2z = 0^{p+j}$.

We claim that $xy^2z \not\in L_3$. Note that $2^{p+j} > 2^p$.

The next smallest string in $L_3$ has length $2^{p+1}$. We must show that $2^{p+j} < 2^{p+1}$. $2^{p+j} < 2^{p+2j} < 2^{p+2j+2} = 2^{p+1}$. It follows that $xy^2z \not\in L_3$ and $L_3$ is not regular.
11) $L_1$ is regular. Here is a DFA that accepts it:

![DFA Diagram]

State $a$ represents that nothing has been read in.
States $b$ and $c$ represent that the string read in so far satisfies the given criteria and started with 0 or 1, respectively.
State $d$ represents a string starting with 0 and ending with 1.
State $e$ represents a string starting with 1 and ending with 0.

Basically, any string starting and ending with the same character is accepted since it contains an equal number of 01 and 10 transitions.

$L_2$ is not regular. We will show that it does not satisfy the pumping lemma. Consider the string $S = (00)^p(11)^p$, where $p$ is the pumping length. $S \in L_2$. Let $S = xyz$ with $x = 0^i, y = 0^j, z = 0^{2p-1-i-j}(11)^p$. Consider $xy^2z = 0^{2p+j-1}$. $xy^2z \notin L_2$ because it has $2p+j-1$ occurrences of 00 and $2p-1$ occurrences of 11. Since $j > 0$, $2p+j-1 > 2p-1$ and $xy^2z \notin L_2$.

It follows that $L_2$ is NOT regular.