1) Let MAX-CLIQUE = \{ <G, k> | G is a graph and its largest clique is of size k \}. If CLIQUE is in P, prove that MAX-CLIQUE is ALSO in P. (Namely, given a black box that solves the CLIQUE decision problem in polynomial time, design a solution to MAX-CLIQUE in polynomial time.

2) Why are 2 x 3 windows necessary in the proof of the Cook-Levin theorem?

3) Using the polynomial time reduction show in the text in 7.4, create the graph that the reduction produces for the following boolean formula:

\[(\overline{a} \lor b \lor \overline{c}) \land (a \lor \overline{b} \lor c) \land (\overline{a} \lor \overline{b} \lor c) \land (a \lor \overline{b} \lor \overline{c})\]

4) Show a polynomial time reduction from 4-SAT to 3-SAT, where 4-SAT represents satisfiability for boolean formulas with four literals in each clause instead of 3, and the formula is still in conjunctive normal form. (Namely, show how to transform a boolean formula in 4-SAT form into an equivalent formula in 3-SAT form such that the input formula is satisfiable if and only if the output formula is.)

5) Let DOUBLE-SAT = \{ <\varphi> | \varphi has at least two satisfying assignments \}. Show that DOUBLE-SAT is NP-Complete by giving a reduction from 3-SAT to DOUBLE-SAT.