1. Design a DFA to recognize the language consisting of strings over \( \{a, b\} \) that do not contain the substring \( abb \).

   **Solution** See page 3 for state diagram.

2. Let \( |w|_x \) denote the number of occurrences of the symbol \( x \) in the string \( w \). Let \( L \) be the language consisting of strings over \( \{a, b\} \) for which either \( |w|_a = 3 |w|_b \) or \( |w|_b > |w|_a \).

   Prove or disprove that \( L \) is regular.

   **Solution**

   Proof by contradiction. Assume \( L \) is a regular language. By the Pumping Lemma, there is a constant \( n \) associated with \( L \).

   (a) Choose the string \( s = a^{3n}b^n \). Note that \( s \in L \) because \( |w|_a = 3 |w|_b \), and \( |s| = 4n \geq n \).

   (b) Choose the partition \( s = xyz \) such that \( x = \epsilon, y = a^n, z = a^{2n}b^n \). Note that \( |y| = n \geq n \).

   (c) In any possible division \( y = uvw \), we must have \( v = a^m \), where \( 0 < m \leq n \).

   (d) Choose \( i = 2 \). Then \( xuv^2wz = xuv^2wz = a^{3n+m}b^n \). Because \( |xuv^2wz|_a = 3n + m > 3n = 3|xuv^2wz|_b \) and \( |xuv^2wz|_b = n < 3n + m = |xuv^2wz|_a \), we have \( |xuv^2wz|_a \neq 3|xuv^2wz|_b \) and \( |xuv^2wz|_b < |xuv^2wz|_a \). Thus \( xuv^2wz \notin L \).

   This is a contradiction. Therefore \( L \) is not a regular language.

3. Construct an NFA that recognizes the language \((ab \cup (aa)^*bb)^*\).

   **Solution** See page 3 for state diagram.

4. DFA to regular expression conversion.

   **Solution** \( 10^*1(00^*1 \cup 1(0 \cup 10^*)^*)^* \). See page 4 for diagrams of the conversion procedure.
5. NFA to DFA construction.

Solution See page 3 for state diagrams.

The NFA is \( (\{A, B, C, D\}, \{0, 1\}, \delta_{NFA}, D, \{C\}) \), where \( \delta_{NFA} \) is given by

\[
\begin{array}{c|ccc}
\delta_{NFA} & 0 & 1 & \epsilon \\
\hline
A & \{A\} & \{D\} & \emptyset \\
B & \{C\} & \{B\} & \{A\} \\
C & \{B\} & \emptyset & \emptyset \\
D & \emptyset & \{A, C\} & \emptyset \\
\end{array}
\]

Using the construction in Theorem 1.19, the constructed DFA is

\[
(\mathcal{P}(\{A, B, C, D\}), \{0, 1\}, \delta_{DFA}, \{D\}, \{S \subseteq \{A, B, C, D\} \mid C \in S\}),
\]

where \( \delta_{DFA} \) is given by

\[
\begin{array}{c|cc}
\delta_{DFA} & 0 & 1 \\
\hline
\{D\} & \emptyset & \{A, C\} \\
\{A, C\} & \{A, B\} & \{D\} \\
\{A, B\} & \{A, C\} & \{A, B, D\} \\
\{A, B, D\} & \{A, C\} & \{A, B, C, D\} \\
\{A, B, C, D\} & \{A, B, C\} & \{A, B, C, D\} \\
\{A, B, C\} & \{A, B, C\} & \{A, B, D\} \\
\end{array}
\]

(Note that, for the sake of brevity, unreachable states and sink states are left out of this table and the corresponding state diagram.)
1. The strings over \{a,b\} that do not contain the substring abb.

![Diagram of NFA](image)

3. NFA that recognizes the language (ab U (aa)*(bb))*.

![Diagram of NFA](image)

5. NFA to DFA construction.

![Diagram of NFA and DFA](image)
4. DFA to regular expression conversion.