Present a transition diagram for a DFA that recognizes the set of binary strings that starts with a 1 and, when interpreted as entering the DFA most to least significant digit, each represents a binary number that is divisible by seven. Thus, 111, 1110 and 10101 are in the language, but 110, 1001 and 11000 are not.

Construction:
I can do on board, but this is a simple variant of one I already did. It must have an extra start state to guarantee a 1 is leftmost symbol. There are then seven more states, labeled $M_0$ to $M_6$ for the value mod 7. $M_0$ is final, $M_i$ goes to $M_{2i \mod 7}$ on a 0 and to $M_{2i+1 \mod 7}$ on a 1. The start state goes to $M_1$. 
a.) Present a transition diagram for an NFA for the language associated with the regular expression \((1101 + 110 + 11)^*\). Your NFA must have no more than four states.

b.) Use the standard conversion technique (subsets of states) to convert the NFA from (a) to an equivalent DFA. Be sure to not include unreachable states. Hint: This DFA should have no more than seven states.
Using DFA's (not any equivalent notation) show that the Regular Languages are closed under \( \text{Max} \), where \( \text{Max}(L) = \{ w \mid w \in L, \text{ and w is not the proper prefix of any other string in L} \} \).

This means that \( w \in \text{Max}(L) \) iff \( w \in L \) and for no \( x \neq \lambda \) is \( wx \) in \( L \). Said a third way, no extension of \( w \) is also in \( L \).

Let \( A = (Q, \Sigma, \delta, q_0, F) \) be a DFA such that \( L = L(A) \).

Define \( A_{\text{MAX}} = (Q, \Sigma, \delta, q_0, F') \), where \( F' = \{ f \mid f \in F \text{ and no member of F, including f, is in Reachable}^+(f) \} \), and \( \text{Reachable}^+(q) = \{ s \mid \delta^*(q, x) = s, \text{ for some } x, \text{ where } |x|>0 \} \).

\[
L(A_{\text{MAX}}) = \{ w \mid \delta^*(q_0, w) \in F \text{ but for no } x, |x|>0, \text{ does } \delta^*(q_0, wx) \in F \}
\]

This is just the definition of \( \text{MAX}(L) \) recast in terms of the behavior of its accepting DFA and so our construction works as desired.