Present a language \( L \) over \( \Sigma = \{a\} \) where \( L^3 = L^4 \) but \( L \neq L^2 \) and \( L^2 \neq L^3 \)

Note: \( L^k = \{ x_1x_2...x_k \mid x_1,x_2,...,x_k \in L \} \)

Proof:
Consider \( L = \{a\}^* - \{aa, aaa\} \)

\( L^2 = \{a\}^* - \{aaa\} \) since the presence of the empty string in \( \{a\}^* \) means all strings in \( L \) are in \( L^2 \). Additionally, \( aa = a^* a \) and so \( aa \) is in \( L^2 \) but \( aaa \) is not since it cannot be formed from any pair of members in \( L \)

\( L^3 = \{a\}^* \) since the presence of the empty string in \( \{a\}^* \) means all strings in \( L^2 \) are in \( L^3 \)
Additionally, \( aaa = aa^* a \) and so \( aaa \) is in \( L^3 \)

\( L^3 = L^4 \) since \( L^3 \) is already \( \{a\}^* \) and so nothing new can be created and the presence of the empty string in \( \{a\}^* \) means all in \( L^3 \) are in \( L^4 \)