1. (50 points total – 10 points each)
Given the relational schemas and instances shown below, construct relational algebra queries, using only the five fundamental operators, that will correctly produce the results for each of the queries (a)-(e).

\( S = \text{STUDENTS}(s\#, \text{name, age, major, gpa, hours\_completed}) \)
\( C = \text{COURSES}(c\#, \text{term, name, dept, enrollment}) \)
\( P = \text{PROFESSORS}(p\#, \text{name, dept, yrs\_teaching, area}) \)
\( TA = \text{TAKES}(s\#, c\#, \text{term, grade}) \)
\( TE = \text{TEACH}(p\#, c\#, \text{term}) \)

(a) List the s# and name of those students who took COP4710 in Fall 2013 term.

\[ r_1 = S \times TA \]
\[ r_2 = \sigma(S.s\# = TA.s\# \text{ AND } TA.c\# = \text{"COP4710" AND } TA.\text{term="Fall2013"})(r_1) \]
\[ r_3 = \pi(s\#, \text{name})(r_2) \]

or

\[ r_1 = \sigma(TA.c\# = \text{"COP4710" AND } TA.\text{term="Fall2013"})(TA) \]
\[ r_2 = \pi(TA.s\#)(r_1) \]
\[ r_3 = S \times r_2 \]
\[ r_4 = \sigma(S.s\# = TA.s\#)(r_3) \]
\[ r_5 = \pi(s\#, \text{name})(r_4) \]

(b) List the s# and name for those students who have not taken COP4710.

\[ r_1 = \sigma(TA.c\# \neq \text{"COP4710"})(TA) \]
\[ r_2 = \pi(TA.s\#)(r_1) \text{ //students in some course which is not cop 4710} \]
\[ r_3 = \sigma(TA.c\# = \text{"COP4710"})(TA) \]
\[ r_4 = \pi(TA.s\#)(r_3) \text{ //students in cop 4710 in some term} \]
\[ r_5 = r_2 - r_4 \quad \text{//s\# never in cop 4710} \]
\[ r_6 = r_5 \times S \]
\[ r_7 = \sigma_{(r_5.s\# = S.s\#)}(r_6) \]
\[ r_8 = \pi_{(s\#, \text{name})}(r_7) \]

(c) List the p\# and name of every professor who has taught the student with s\# = 12.

\[ r_1 = \sigma_{(s\# = "12")}(TA) \]
\[ r_2 = TE \times r_1 \]
\[ r_3 = \sigma_{(TE.c\# = TA.c\# \text{ AND } TE.\text{term} = TA. \text{term})}(r_2) \]
\[ r_4 = P \times r_3 \]
\[ r_5 = \sigma_{(P.p\# = TE.p\#)}(r_4) \]
\[ r_6 = \pi_{(p\#, \text{name})}(r_5) \]

(d) List the p\# and names of professors who have only taught students who are CS majors.

\[ r_1 = \sigma_{(\text{major} = "CS")}(S) \]
\[ r_2 = \pi_{(s\#)}(r_1) \]
\[ r_3 = TA \times r_2 \]
\[ r_4 = \sigma_{(TA.s\# = S.s\#)}(r_3) \]
\[ r_5 = TE \times r_4 \]
\[ r_6 = \sigma_{(TE.c\# = TA.c\# \text{ AND } TE.\text{term} = TA. \text{term})}(r_5) \]
\[ r_7 = P \times r_6 \]
\[ r_8 = \sigma_{(P.p\# = TE.p\#)}(r_7) \]
\[ r_9 = \pi_{(p\#)}(r_8) \]
\[ r_{10} = \sigma_{(\text{major} \neq "CS")}(S) \]
\[ r_{11} = \pi_{(s\#)}(r_{10}) \]
\[ r_{12} = TA \times r_{11} \]
\[ r_{13} = \sigma_{(TA.s\# = S.s\#)}(r_{12}) \]
\[ r_{14} = TE \times r_{13} \]
\[ r_{15} = \sigma_{(TE.c\# = TA.c\# \text{ AND } TE.\text{term} = TA. \text{term})}(r_{14}) \]
\[ r_{16} = P \times r_{15} \]
\[ r_{17} = \sigma_{(P.p\# = TE.p\#)}(r_{16}) \]
\[ r_{18} = \pi_{(P.p\#)}(r_{17}) \]
\[ r_{19} = r_9 - r_{18} \]
\[ r_{20} = P \times r_{19} \]
\[ r_{21} = \sigma_{(P.p\# = r_{20}.p\#)}(r_{20}) \]
\[ r_{22} = \pi_{(p\#, \text{name})}(r_{21}) \]

(e) List the s# for those students who have taken every course.

\[ r_{1} = \pi_{(s\#, \text{c\#})}(TA) \quad // \text{all s\#,c\# pairs for students who have taken a course} \]
\[ r_{2} = \pi_{(c\#)}(C) \quad // \text{all c\#s} \]
\[ r_{3} = \pi_{(s\#)}(r_{1}) \quad // \text{all s\# for students who have taken a course} \]
\[ r_{4} = r_{2} \times r_{3} \quad // \text{all possible s\#,c\# pairs for students who have taken a course} \]
\[ r_{5} = r_{4} - r_{1} \quad // \text{leaves only pairs that didn't actually exist in TA} \]
\[ r_{6} = \pi_{(s\#)}(r_{5}) \quad // \text{get just s\# from pairs that didn't really exist in TA} \]
\[ r_{7} = r_{3} - r_{6} \quad // \text{leaves only s\# who have taken every course} \]

2. (50 points total – 10 points each)
Produce relational algebra query expressions for each of the queries (a)-(e) in Question 1 above, but this time use the redundant relational algebra operators whenever possible.

(a) List the s# and name of those students who took COP4710 in Fall 2013 term.

\[ r_{1} = S \ast TA \quad // \text{join on s\#} \]
\[ r_{2} = \sigma_{(TA.c\# = "COP4710" \text{ AND TA.term}="Fall2013")}(r_{1}) \]
\[ r_{3} = \pi_{(s\#, \text{name})}(r_{2}) \]

or

\[ r_{1} = \sigma_{(TA.c\# = "COP4710" \text{ AND TA.term}="Fall2013")}(r_{1}) \]
\[ r_{2} = S \ast r_{1} \]
\[ r_{3} = \pi_{(s\#, \text{name})}(r_{2}) \]

(b) List the s# and name for those students who have not taken COP4710.

\[ r_{1} = \sigma_{(TA.c\# \neq "COP4710")}(TA) \]
\[ r_{2} = \pi_{(TA.s\#)}(r_{1}) \quad // \text{students in some course which is not cop 4710} \]
\[ r_{3} = \sigma_{(TA.c\# = "COP4710")}(TA) \]
\[ r4 = \pi_{(\text{TA}, \text{s#})}(r3) \] //students in cop 4710 in some term
\[ r5 = r2 - r4 \] //s# never in cop 4710
\[ r6 = r5 \ast S \] //join on s#
\[ r7 = \pi_{(\text{s#}, \text{name})}(r6) \]

(c) List the p# and name of every professor who has taught the student with s# = 12.

\[ r1 = \sigma_{(\text{s#} = \text{"12"})}(\text{TA}) \]
\[ r2 = \text{TE} \ast r1 \] //join on c# and term
\[ r3 = \text{P} \ast \text{TE} \] //join on p#
\[ r4 = \pi_{(\text{p#}, \text{name})}(r3) \]

(d) List the p# and names of professors who have only taught students who are CS majors.

\[ r1 = \sigma_{(\text{major} = \text{"CS"})}(\text{S}) \]
\[ r2 = \pi_{(\text{s#})}(r1) \]
\[ r3 = \text{TA} \ast r2 \] //join on s#
\[ r4 = \text{TE} \ast r3 \] //join on c# and term
\[ r5 = \text{P} \ast r4 \] //join on p#
\[ r6 = \pi_{(\text{p#})}(r5) \]
\[ r7 = \sigma_{(\text{major} \neq \text{"CS"})}(\text{S}) \]
\[ r8 = \pi_{(\text{s#})}(r7) \]
\[ r9 = \text{TA} \ast r8 \] //join on s#
\[ r10 = \text{TE} \ast r9 \] //join on c# and term
\[ r11 = \text{P} \ast r10 \] //join on p#
\[ r12 = \pi_{(\text{p#})}(r11) \]
\[ r13 = r6 - r12 \]
\[ r14 = \text{P} \ast r13 \] //join on p#
\[ r15 = \pi_{(\text{p#}, \text{name})}(r14) \]
(e) List the s# for those students who have taken every course.

\[ r1 = \pi_{(s\#, c\#)}(TA) \]
\[ r2 = \pi_{(c\#)}(C) \]
\[ r3 = r1 \div r2 \]

3. (50 points total – 10 points each)
Produce tuple relational calculus expressions for each of the queries (a)-(e) in Question 1 above.

(a) List the s# and name of those students who took COP4710 in Fall 2013 term.

\[ \{ x.s\#, x.name \mid x \in S \text{ and } \exists y \in TA (y.term = "Fall 2013" \text{ and } y.c\# = "COP4710" \text{ and } y.s\# = x.s\# ) \} \]

(b) List the s# and name for those students who have not taken COP4710.

\[ \{ x.s\#, x.name \mid x \in S \text{ and not } \exists y \in TA (y.c\# = "COP 4710" \text{ and } y.s\# = x.s\# ) \} \]

(c) List the p# and name of every professor who has taught the student with s# = 12.

\[ \{ x.p\#, x.name \mid x \in P \text{ and } \exists y \in TE (y.p\# = x.p\# \text{ and } \exists z \in TA (z.c\# = y.c\# \text{ and } z.term = y.term \text{ and } z.s\# = "12" ) ) \} \]

(d) List the p# and names of professors who have only taught students who are CS majors.

\[ \{ x.p\#, x.name \mid x \in P \text{ and not } \exists y \in TE (y.p\# = x.p\# \text{ and } \exists z \in TA (z.c\# = y.c\# \text{ and } z.term = y.term \text{ and } \exists w \in S (w.s\# = z.s\# \text{ And } w.major \neq "CS") ) ) \} \]

(e) List the s# for those students who have taken every course.

\[ \{ x.s\# \mid x \in S \text{ and not } \exists y \in C (\text{not } \exists z \in TA (z.c\# = y.c\# \text{ and } x.s\# = z.s\# )) \} \]
or –

\{ x.s# \mid x \in S \text{ and } \forall y \in C \ ( \exists z \in TA ( y.c# = z.c# \text{ and } z.s# = x.s# ) ) \}