Review Questions for Exam 1

1. Transform the following infix expression into its equivalent postfix expression using a stack. Trace the state of the operator stack as each character of the infix expression is processed. Show the contents of the operator stack at the indicated points in the infix expressions (points A, B and C).

   \[ R + P \times Q / T - ( F - J + K ) \times N \]

   Resulting Postfix Expression: ____________________________

2. Evaluate the following postfix expression, showing the values in the stack at each indicated point in the Postfix string (points A, B, and C).

   \[ 8 \ 5 \ * \ 4 \ 6 \ + \ / \ 2 \ 8 \ * \ - \ 4 \ 5 \ + \ * \]

   Resulting Postfix Expression: ____________________________
The Final value of the expression is ________________

3. Express using summations the number of arithmetic operations executed by the following code in lines 6 and 9. You don’t have to evaluate the summations.

```java
1   sum =0;
2   a =0; b= 5;
3   for ( i=0, i <n; i++) {
4       for (j=5; j<=m; j++) {
5           for ( k = 1; k < j;  k++){
6               f = sum + b +k ;
7           }
8       }
9       a = (a+ i * 7)/ 4;
10  }
```

4. Evaluate the following using summation rules. No points will be awarded if the summation rules are not used. Show all of your work to get credits on this question.

\[
\sum_{i=0}^{n} 2i \\
\sum_{j=1}^{n} 5 \\
\sum_{i=0}^{n} 4i - 2
\]
5. a) Consider the circular implementation of array based queue, that we discussed in the class, with size = 35. How many elements can be added to the queue in following situations?

i) front = –1, rear = –1

ii) front = 0, rear = 16

iii) front = 6, rear = 0

iv) front = 16, rear = 16

v) front = 34, rear = 0

5. b) [10 pts] Assuming that there are M items in a stack, what is the time complexity of performing following operations on the array based stacks that we have studied in the class. Express your result in terms of big-O notation.

i) one pop operation O(1)

ii) n push operations O(n)

6. a) [6 pts] For the following function write the recurrence relation to indicate the total number of operations T(n). You are not required to solve it.

```c
int operation( int x, int n )
{
    if (n < 0)
        return 5;
    else return ( x – 3 + operation( x, n – 3 );
}
```
6. b) Trace the following function for
   i) \( N = 3 \).
   ii) \( N = 5 \)

   ```c
   int puzzle (int N)
   {
     if (N == 1) return 1;
     if (N % 2 == 0)
       return (1 + puzzle(N/2));
     else
       return (2 + puzzle(3*N+1));
   }
   ```

   Answer:
   i) \( \text{puzzle}(3) = 10 \)

   ii) \( \text{puzzle}(5) = 7 \)

6c) Write a recursive function which returns the number of integers greater than 50 in the array \( dd \) containing \( k \) elements.

**More problems:**

7. Develop an algorithm to find the largest element in a two dimensional array.

8. Develop an algorithm to reverse the digits of a given integer. Thus if the input integer was 567, the algorithm should produce the output 765.

9. Find the number of multiplication operations in the following pseudo code segments:

    a) \( t = 1; \)
        \[
        \text{for } j = 1 \text{ to } N \{ \\
        \quad \text{for } i = 1 \text{ to } k \\
        \quad \quad s = t * j; \\
        \}
        \]

    b) \( s = 5; \)
\[ t = 10; \]
\[ k = 2; \]
\[ \text{for } j = 1 \text{ to } 40 \{ \]
\[ \quad \text{for } i = 1 \text{ to } t \]
\[ \quad \text{ } s = s + t \times i \times 8; \]
\[ \} \]

\[ \text{c) } s = 5; \]
\[ \text{for } j = 1 \text{ to } 40 \{ \]
\[ \quad s = s \times 2; \]
\[ \quad \text{for } i = 1 \text{ to } 20 \]
\[ \quad s = t \times j \times 8; \]
\[ \} \]

10. Algorithm A runs in \( O(N^3) \) time. For an input size of 10, the algorithm runs in 7 milliseconds. For another input size, the algorithm takes 189 milliseconds. What was that input size?

11. Algorithm A runs in \( O(\log_2 N) \) time, and for an input size of 16, the algorithm runs in 28 milliseconds, how long can you expect it to take to run on an input size of 64?

12. Suppose each of the following expressions represent the number of logical operations in an algorithm as a function of \( n \), the size of the list being manipulated. For each expression determine the dominant term and classify the algorithm in big-O terms.

a) \( 4n - 30 \)

b) \( 10n + 20 \log_2 n \)

c) \( 6 \log_2 n + 30 \)

d) \( 5n + 40 \)

e) \( 10n + 2n \log_2 n \)

f) \( 6n + 2(\log_2 n)^2 \)

g) \( 20n^3 + 2n^2 \log_2 n + n^3 \log_2 n + 5n^2 \)

h) 5
13. Given an array A of size n containing elements in random order, an array B of size n containing elements organized in increasing order, a stack C containing n elements, express the worst time complexity of following operations in terms of Big-O. Assume efficient algorithms are being used.

a) to print the last element of array A. ________________

b) to print the last element of array B. ________________

c) to print smallest element of array A. ________________

d) to add a new element to C. ________________

e) to search for a specific element in B ________________