Lecture-8

Haralick’s Edge Detector

Haralick’s Edge Detector

- Fit a bi-quadratic polynomial to a small neighborhood of a pixel.
- Compute analytically second and third directional derivatives in the direction of gradient.
- If the second derivative is equal to zero, and the third derivative is negative, then that point is an edge point.
Haralick’s Edge Detector

Bi-cubic polynomial:
\[ f(x, y) = k_1 + k_2 x + k_3 y + k_4 x^2 + k_5 x y + k_6 y^2 + k_7 x^3 + k_8 x^2 y + k_9 x y^2 + k_{10} y^3. \]

Gradient angle, defined with positive y-axis:
\[ \sin\theta = \frac{k_9}{\sqrt{k_7^2 + k_9^2}}, \]
\[ \cos\theta = \frac{k_7}{\sqrt{k_7^2 + k_9^2}}. \]

Homework

Directional derivative \[ f' = \frac{\partial f}{\partial x} \cos\theta + \frac{\partial f}{\partial y} \sin\theta. \]

Gradient angle, defined with positive x-axis:

Haralick’s Edge Detector

\[ f'_x = \frac{\partial f}{\partial x} \cos\theta + \frac{\partial f}{\partial y} \sin\theta. \]
\[ f'_y = \frac{\partial f}{\partial y} \sin\theta + \frac{\partial f}{\partial x} \cos\theta, \]
\[ f''(x, y) = \frac{\partial^2 f}{\partial x^2} \sin^2\theta + \frac{\partial^2 f}{\partial y^2} \cos^2\theta + 2 \frac{\partial^2 f}{\partial x \partial y} \cos\theta \sin\theta. \]
Haralick’s Edge Detector

\[ f(x, y) = k_1 + k_2x + k_3y + k_4x^2 + k_5xy + k_6y^2 + k_7x^3 + k_8x^2y + k_9xy^2 + k_{10}y^3. \]

\[ x = \rho \sin \theta, \quad y = \rho \cos \theta \]

\[ f_\theta(\rho) = C_0 + C_1\rho + C_2\rho^2 + C_3\rho^3. \]  

\[ C_0 = k_1, \]
\[ C_1 = k_2 \sin \theta + k_3 \cos \theta, \]
\[ C_2 = k_4 \sin^2 \theta + k_5 \sin \theta \cos \theta + k_6 \cos^2 \theta, \]
\[ C_3 = k_7 \sin^3 \theta + k_8 \sin^2 \theta \cos \theta + k_9 \sin \theta \cos^2 \theta + k_{10} \cos^3 \theta. \]

Haralick’s Edge Detector

\[ f_\theta(\rho) = C_0 + C_1\rho + C_2\rho^2 + C_3\rho^3. \]

\[ f'_\theta(\rho) = C_1 + 2C_2\rho + 3C_3\rho^2, \]
\[ f''_\theta(\rho) = 2C_2 + 6C_3\rho, \]
\[ f'''_\theta(\rho) = 6C_3. \]

\[ f''_\theta(\rho) < 0, \text{ we get } 6C_3 < 0, \text{ or } C_3 < 0. \]

\[ f^\theta(\rho) = 2C_2 + 6C_3\rho = 0, \text{ we get } \left| \frac{C_3}{2C_2} \right| < \rho_0. \]
Haralick’s Edge Detector

First order polynomial

\[ f(x, y) = k_0 + k_1 x + k_2 y. \]

9 points give 9 eqs

\[ \begin{align*}
  f_1 &= k_1 + k_2 x_1 + k_3 y_1, \\
  f_2 &= k_1 + k_2 x_2 + k_3 y_2, \\
  &\vdots \\
  f_9 &= k_1 + k_2 x_9 + k_3 y_9.
\end{align*} \]

\[
\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_9 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots \\ 1 & x_9 & y_9 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix},
\]

\[ f = A k. \]

\[ (A^T A)^{-1} A^T f = k, \]

\[ B f = k. \]

Haralick’s Edge Detector

\[ B = (A^T A)^{-1} A^T \text{ is a } 3 \times 9 \text{ matrix} \]

\[
\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_9 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix},
\]

\[ k_3 = b_{11} f_1 + b_{12} f_2 + b_{13} f_3 + b_{14} f_4 + b_{15} f_5 + b_{16} f_6 + b_{17} f_7 + b_{18} f_8 + b_{19} f_9 \]

\[ k_1 = f \ast b_1. \]
Computing coefficients using convolution

\[ f \]

\[ K_1 = \begin{array}{ccc}
\pi & \tau & \beta \\
\sigma & \tau & \gamma \\
\tau & \alpha & \delta
\end{array} \]

\[ b_{11} b_{12} b_{13} \\
\vdots \]

\[ b_{37} b_{38} b_{39} \]
Haralick’s Edge Detector

1. Find $k_1, k_2, k_3, \ldots, k_{10}$ using least square fit, or masks given in Figure 2.8.
2. Compute $\theta$, $\sin \theta$, $\cos \theta$.
3. Compute $C_2, C_3$.
4. If $C_3 < 0$ and $|\frac{\partial f}{\partial x}| < \rho_0$ then that point is an edge point.

Figure 2.9: The steps in Haralick’s Edge Detector.

Comparison of Three Edge Detectors

- Marr-Hildreth
  - Gaussian filter
  - Zerocrossings in Laplacian
- Canny
  - Gaussian filter
  - Maxima in gradient magnitude
- Haralick
  - Smoothing through bi-cubic polynomial
  - Zerocrossings in the second directional derivative, and negative third derivative
Laplacian and the second Directional Derivative and the direction of Gradient

\[ \nabla^2 f = f_{xx} + f_{yy} = f'' + f_n'' \]

\[ f'_n = f_x \cos \theta + f_y \sin \theta \]
\[ f''_n = (f_{xx} \cos \theta + f_{yy} \sin \theta) \cos \theta + (f_{xy} \cos \theta + f_{yy} \sin \theta) \sin \theta \]
\[ f''_n = f_{xx} \cos^2 \theta + f_{yy} \sin^2 \theta + 2 f_{xy} \cos \theta \sin \theta \]
\[ f''_n = f_{xx} \cos^2 \theta + f_{yy} \sin^2 \theta + 2 f_{xy} \cos \theta \sin \theta \]
\[ f''_n = f_{xx} \cos^2 (\theta + 90) + f_{yy} \sin^2 (\theta + 90) + 2 f_{xy} \cos (\theta + 90) \sin (\theta + 90) \]
\[ f''_n = f_{xx} \sin^2 \theta + f_{yy} \cos^2 \theta + 2 f_{xy} \cos \theta \sin \theta \]

Laplacian and the second Directional Derivative and the direction of Gradient

\[ f''_n = f_{xx} \cos^2 \theta + f_{yy} \sin^2 \theta + 2 f_{xy} \cos \theta \sin \theta \]
\[ f''_n = f_{xx} \sin^2 \theta + f_{yy} \cos^2 \theta + 2 f_{xy} \cos \theta \sin \theta \]
\[ \nabla^2 f = f_{xx} + f_{yy} = f''_n + f_n'' \]
Scales

- What should be sigma value for Canny and LG edge detection?
  - Marr-Hildreth:
- If use multiple sigma values (scales), how do you combine multiple edge maps?
  - Spatial Coincidence assumption:
    - Zerocrossings that coincide over several scales are physically significant.

Scale Space

- Apply whole spectrum of scales
- Plot zerocrossings vs scales in a scale-space
- Interpret scale space contours
  - Contours are arches, open at the bottom, closed at the top
  - Interval tree
    - Each interval $I$ corresponds to a node in a tree, whose parent node represents larger interval, from which interval $I$ emerged, and whose off springs represents smaller intervals into which $I$ subdivides.
    - Stability of a node is a scale range over which the interval exits.
Scale Space

- Top level description
  - Iteratively remove nodes from the tree, splicing out nodes that are less stable than any of their parents and off springs
Scale Space

A top level description of several signals using stability criterion.