Problem 1. (35 points) The bode plot for a unity feedback system is shown below:

a). Find the gain crossover frequency.
b). Find the phase crossover frequency.
c). Find the gain margin.
d). Find the phase margin.
e). What is the system type?

Solution: 

a). \( \omega_g = 1 \text{ rad/sec} \).
b). \( \phi = 4 \text{ rad/sec} \).
c). \( GM = 20 \text{ dB} = 10 \) 
   or \( a = 20 \log x = -20 \Rightarrow x = 0.1 \Rightarrow GM = \frac{1}{x} = 10 \).
d). \( \phi = -140^\circ \), \( PM = 180^\circ + \phi = 40^\circ \).
e). slope at low frequency is -20dB/dec, type 1.
Problem 2. (65 points)

For the system shown in the figure, where

\[ G(s) = \frac{K}{(s + 2)(s + 4)(s + 6)} \]

a). Plot the Nyquist diagram for \( K=1 \) and find the negative real axis crossing; (30 points)
b). Is the system stable for \( K=1 \)? (15 points)
b). Use Nyquist criterion to find the range of gain, \( K \), for the closed-loop system to be stable. (20 points)

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\text{Solution: a).} \quad \text{The frequency response is } 1/48 \text{ at an angle of } 0^\circ \text{ degrees at } \omega = 0. \text{ Each pole rotates } 90^\circ \text{ in going from } \omega = 0 \text{ to } \omega = \infty. \text{ Thus, the resultant rotates } -270^\circ \text{ while its magnitude goes to zero. The result is shown below.}
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\text{Substituting } j\omega \text{ into } G(s) = \frac{1}{(s + 2)(s + 4)(s + 6)} = \frac{1}{s^3 + 12s^2 + 44s + 48} \text{ and simplifying, we obtain } G(j\omega) = \frac{(48 - 12\omega^2) - j(44\omega - \omega^3)}{\omega^6 + 56\omega^4 + 784\omega^2 + 2304}. \text{ The Nyquist diagram crosses the real axis when the imaginary part of } G(j\omega) \text{ is zero. Thus, the Nyquist diagram crosses the real axis at } \omega^3 = 44, \text{ or } \omega = \sqrt[3]{44} = 6.63 \text{ rad/s. At this frequency } G(j\omega) = -\frac{1}{480}.
\]

b). \( Z = N + P = 0 \Rightarrow \text{system stable.} \)

C). \( K = \frac{-1}{-480} = 0.480 \Rightarrow \text{system stable for } K < 480 \).