Exam #2
EEL 3657 (Spring 2003)

Please show all work for partial credit. (100 points total)

Problem 1. (50 points) For a unity feedback system shown below,

\[ R(s) + E(s) \quad G(s) \quad C(s) \]

where \( G(s) = \frac{K}{s(s+2)^2} \), sketch the root locus and find:

1. asymptotes;
2. break-away/break-in points;
3. jw axis crossings;
4. K value at the jw axis crossings.

Solution:

1. \( \theta_a = \frac{-4 + 0}{3 - 0} = -\frac{4}{3} \),
   \( \phi_a = \frac{180^\circ + 360^\circ l}{3} \) \( l = 0, 1, 2 \) 
   \( = 60^\circ, 180^\circ, 300^\circ \).

2. \( \frac{dk}{ds} = 0 \Rightarrow 3s^2 + 8s + 4 = 0 \)
   \[ s = -2, -\frac{2}{3} \].

3. \( CE: s^3 + 4s^2 + 4s + k = 0 \)
   Substitute \( s = jw \):
   \[ -jw^3 - 4w^2 + j4w + k = 0 \]
   \[ s - 4w^2 + k = 0 \]
   \[ -w^3 + 4w = 0 \] \( \Rightarrow \) \( w = 0, 2 \)
   \( k = 0, 16 \).
   So jw axis crossing is at \( \pm j2 \).

4. \( k = 16 \).
Problem 2. (50 points) The block diagram of a linear control system is shown:

where \( G_p(s) = \frac{1}{(s+3)(s+6)} \), \( H(s) = 1 \), \( G_c(s) = \frac{K(s+a)}{s} \), \( (G_c(s) \) is a PI Controller) 

1). Let \( a = 0.1 \), find the dominate closed-loop poles with the damping ratio \( \xi = 0.707 \).

2). Find \( K \) at the above determined poles.

3). What is the closed-loop steady-state error for a unit step input?

\[
G_c G_p = \frac{K(s+0.1)}{s(s+3)(s+6)}
\]

\[
\xi = 0.707 \implies 0 = \sin^{-1} \frac{8}{3} = 45^\circ
\]

Assume the dominate pole is at \(-x + jx\):

Use angle condition:

\[
2\phi_1 - \phi_2 - \phi_3 = -180^\circ
\]

\[
(180^\circ - \tan^{-1} \frac{x}{x-0.1}) - 135^\circ - \tan^{-1} \frac{x}{3-x} - \tan^{-1} \frac{x}{6-x}
\]

\[
\implies x = 4.494
\]

So the dominate closed-loop poles are at \(-4.494 \pm 4.494i\).

2). Use magnitude condition:

\[
K = \left| \frac{1.51 \cdot 15 + 61}{15 + 0.1} \right| = 22.7
\]

3). \( K_p = \lim_{s \to 0} C(s) = \infty \), \( \epsilon_{ss} = \frac{1}{1+K_p} = 0 \).